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PROBLEMS

IN

MACHINE DESIGN.

BY

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PREFACE.

THE object of the Author, in writing this book, is to supply Engineering Students preparing for the Honours Stages of the Science and Art and Technological Examinations in Machine Construction and Mechanical Engineering with a text-book in which there is not merely a collection of formulæ whose use is but little explained, but which contains examples showing how such formulæ may be applied to solve problems in machine design. To all those who are content to copy the designs of others, or who are possessed of that intuition and experience which enable them to dispense with calculation, we do not recommend this book; but there are many others to whom we hope it may be of some service. The book is incomplete for several reasons: firstly, because we did not wish to introduce into it what had been already completely discussed in other text-books on Machine Design; secondly, because we hope to deal with other matter, such as the design of complete machines, in another volume; and thirdly, since we need time for the completion of calculations on other subjects. We are indebted to the writings of Professor Dwelshauvers-Déry for much in the chapter on "Governors," and to Mr. Young for most of the chapter on "Springs." In the report of the Examiners of the Science and Art Department in Machine Construction, for the year 1892, some complimentary remarks were made as to the instruction given by the Author to his Honours Stage Class. We mention this, as it is an additional reason why this book should be published.

C. H. I.

Rutherford College,
Newcastle-on-Tyne.

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PROBLEMS IN MACHINE DESIGN.

CHAPTER I.

GRAPHIC AND OTHER METHODS OF FINDING LONGITUDINAL STRESSES IN FRAMEWORK STRUCTURES.

If three forces, fig. 1, act at a point O, and are in equilibrium, they can be represented in magnitude and direction by the sides of a triangle *b c a*, taken in order. It is frequently convenient to use the method of areas shown here, the force lying between the areas B and C being called the force *b c*, and so on. Also if any number of forces, fig. 3, act at a point, and are in equilibrium, they can be represented by a polygon, whose sides are parallel and proportional to these forces, the line *a b* representing the force between the areas A and B. If, then, at any point of a framework structure, such as a crane, we know the magnitudes of all the longitudinal stresses—*i.e.*, the stresses along the members, except two, and if we know the directions of all of them, we can find the two unknown forces by completing the polygon. Thus, if A B, B C, C D, fig. 3, are known, we can draw the three sides of the polygon corresponding to these, fig. 4, and by making *d e*, *e a*, fig. 4, parallel to D E, E A, fig. 3, whose directions are known, we can find the magnitudes of the two unknown forces.

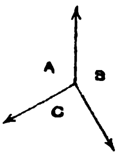


FIG. 1.

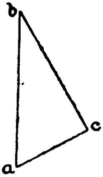


FIG. 2.

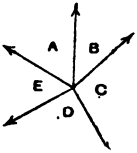


FIG. 3.

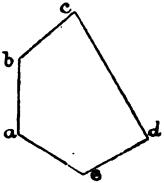


FIG. 4.

Fig. 5 shows a wall crane, capable of carrying a load of 1½ ton, at 110 in. from A. We shall apply the above to find the stresses in the various members. E is the intersection

of C D and A B produced, and as we are not considering bending stresses in A B, the load W may be divided into two parts, $\frac{W.\,BE}{AE}$ at A, and $\frac{W.\,AB}{AE}$ at E.

$$AE = \frac{1500}{11}\text{ in.},$$

and

$$AB = \frac{1210}{11}\text{ in.}$$

So that we have at E—

$$F_1 = \frac{1210}{1500} \times 1\tfrac{1}{2}\text{ ton},$$

$$= 1{\cdot}21\text{ ton.}$$

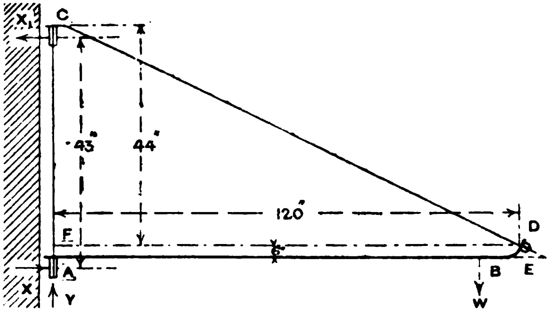


Fig. 5.

Upon the triangle C E A, the forces Y, X, X_1, and W act—

$$Y = W,$$

$$X = X_1,$$

and the couple $W.\,AB = X \times 43\text{ in.}$

$$\therefore\ X = \frac{1{\cdot}5 \times 110}{43} = 3{\cdot}83\text{ tons.}$$

In fig. 6 the triangle C A E is again drawn, and forces, as shown, placed at the angles. G is the central area, and it will be noticed that it is necessary to pass the line of action of a force, or to pass over a side of the triangle, to proceed from one area to another. Now, to draw the diagram of stresses, we take *h m* equal to F_1, and draw *m g*, *g h* parallel to A E, E C, which must now be spoken of as the members M G, G H. It is clear that the stress in M G is compressive, because it exerts a force to the right, at E; we therefore put arrows, as shown, upon M G. The stress in G H is tensile, and this is shown also by arrows, pointing inwards, these representing the directions of the forces exerted by the bar upon the end joints. It will be noticed that we have travelled round the point E in the same direction as that of the hands of a watch, when drawing the triangle *h m g*. Having chosen the direction, we must adhere to it for all the points of the structure. We next draw the triangle *h g k*, to represent the forces at C, showing that the force exerted by G K acts upwards at C. The arrows marked on G K show that it is in compression. Lastly, for A, the rectangle *k g m l* represents the four forces at that point. It must be noted that *h k* and *k l* are not equal to X, nor is *l m* equal to Y.

The above reasoning may not be very simple, but it can now be seen how easy it is to apply this "Method of Areas" to determine the stresses. We draw *h m* parallel and equal

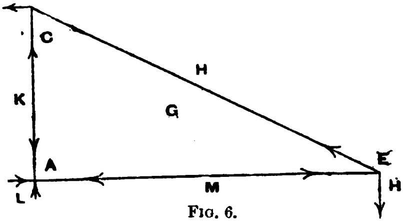


Fig. 6.

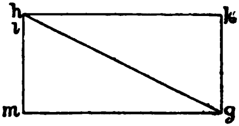


FIG. 7.

to F_1, and *m g*. *h g* parallel to M G and H G, then *g k* and *h k* parallel to G K and H K, and lastly *k l* and *m l* parallel to K L and M L, so that our object is attained without any mental exertion whatever.

Fig. 8 is the side elevation of a sheer legs. A C, A C^1 each = 110 ft., A B = 151½ ft.; the perpendicular from B on C C^1 is 72½ ft., and C C^1 is 42 ft. The load carried is 80 tons, and the weight of each front leg is 9 tons, while that of the back leg is 15½ tons. There is a screw which, passing through a nut at B, moves it in and out, the axis of the screw being the perpendicular from B on C C^1. As we are not dealing with bending stresses, we may suppose these weights transferred to A, B, and C C^1, so that at A we have 96¾ tons, at B 7¾ tons, and at C C^1 9 tons. Fig. 8 is then divided up into areas, as shown, and *f d* is drawn vertically downwards to represent 96¾ tons, and *d e*, *f e* are parallel to D E and F E. It is clear, then, that the former stress is compressive, and the latter tensile. The two remaining lines *e g*, *g f* are next drawn. Then *g e* is the pull on the screw ; *g f*, less $7\frac{3}{4}$ tons, is the actual downward component of the force at B ; and *g d*, plus 9 tons, the vertical component of the resultant ot the two forces acting on the front legs at C C^1. To find the actual stresses on the front legs, we draw fig. 10. To represent the actual sizes of A C, A C^1, divide it into the areas H, K, L, M, and make *l k*, fig. 11, equal to *d e*, fig. 9 ; then *k h*, *h l* are the stresses in A C^1, A C, and *h m*, drawn horizontally, gives us *l m*, *m k*, each of which must be increased by $4\frac{1}{2}$ tons to give the actual value of the vertical forces at C and C^1. This construction will give—

$$\begin{aligned} de &= 171 \text{ tons}, & fe &= 92 \text{ tons}, \\ ge &= 70 \text{ tons}, & fg &= 59 \text{ tons}, \\ gd &= 155\tfrac{3}{4} \text{ tons}, & hl &= 87{\cdot}5 \text{ tons}, \\ hm &= 17 \text{ tons}. \end{aligned}$$

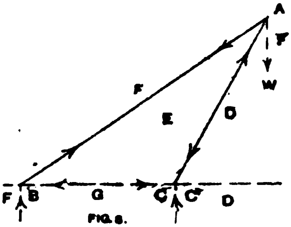


FIG. 8.

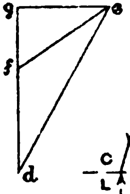


FIG. 9.

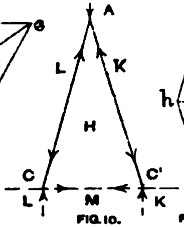


FIG. 10.

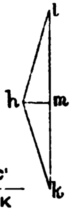


FIG. 11.

In the above we have neglected the tension of the chain, which very nearly coincides with A B. The mechanical advantage is 6 tons, and therefore the tension of the chain is $13\frac{2}{3}$ tons, which reduces the tension in the back leg to $78\frac{1}{3}$ tons, and the horizontal pull on the screw at B is—

$$ge \times \frac{78\frac{1}{3}}{92} = 59{\cdot}6 \text{ tons.}$$

Fig. 12 shows the outline of a platform crane, and fig. 13 the corresponding stress diagram. Each line in fig. 13 is parallel to the corresponding member or force in fig. 12. Thus *e b* is parallel to the member between the areas E and B. Fig. 13 is drawn thus : Make *e d* equal to W, and draw *d a*, *e a*, *a b*, *e b*, *b c*, *e c*, *c g*, *d g*, and *e f ;* by proceeding in this order the figure is readily drawn. To find whether any stress is tensile or compressive, we proceed as follows : In drawing the triangles or polygons for any point, we have decided to proceed round it in the same direction as that of the hands of a watch. Thus at the point X, fig. 12,

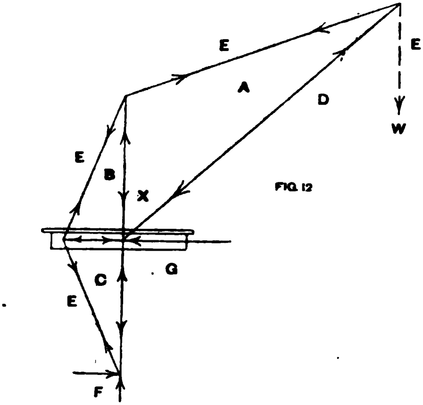


FIG. 12

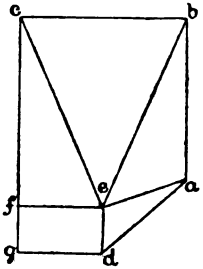


FIG. 13

the polygon is *b a d g c*, so that *b a* acts downwards at X, *a d*, *d g* to the left, *g c* upwards, and c *b* to the right ; therefore the arrows must be placed as shown in fig. 12.

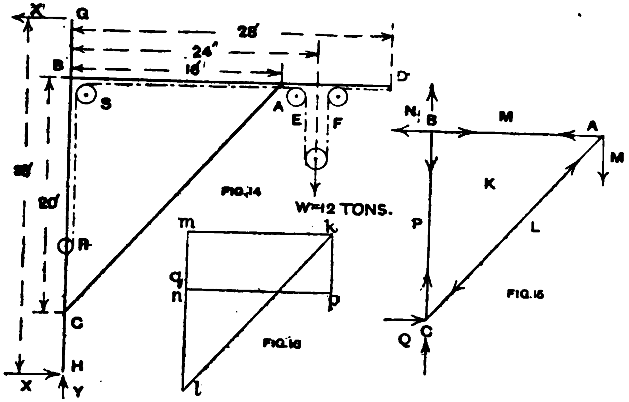


FIG. 14

FIG. 15

FIG. 16

Fig. 14 shows the outline of a well-known type of crane much used in forges and foundries. The weight of the jib

B D is 1½ ton, of the strut A C 1 ton, and of G H 2 tons. The pulleys E F are carried by the traveller or monkey, and can be moved along B D. We see at once that

$$Y = 16\tfrac{1}{2} \text{ tons.}$$

$$X = X^1 = \frac{12 \times 24 + 1\tfrac{1}{2} \times 14 + 8}{28} = 11{\cdot}3 \text{ tons,}$$

assuming that the centres of gravity of B D and A C are at their middle points. To find the longitudinal stresses, W may be replaced by upward and downward forces F_1, F_2 at B and A.

$$F_1 = \frac{W \times 8}{16} = 6 \text{ tons};$$

$$F_2 = \frac{W \times 24}{16} = 18 \text{ tons.}$$

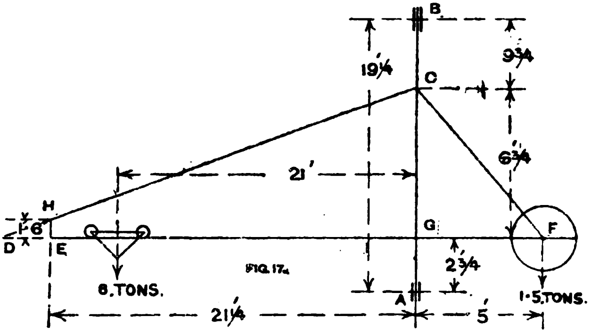


FIG. 17.

The load of the jib may be replaced by two downward forces F_3, F_4 at A and B.

$$F_3 = 1\tfrac{1}{2} \times \frac{14}{16} = 1{\cdot}3125 \text{ ton};$$

$$F_4 = 1\tfrac{1}{2} \times \frac{2}{16} = {\cdot}1875 \text{ ton.}$$

The load of the strut A C adds a ½ ton to C and A.

Thus we have at A, fig. 15, a force *m l* of 19·8125 tons; at B the force *n m* is 5·8125 tons. Fig. 16 shows the stresses, and since those in A B, B C are tensile, they will be each reduced by 6 tons, the compressive force exerted by each part of the chain E S R. That in B C must again be increased by the weight of the part GC above C, which is nearly 2 tons.

It frequently happens that the above method of areas is not the shortest way of solving these problems. Fig. 17 is an outline sketch of a hydraulic ingot stripping crane, capable of lifting a load of 6 tons; B represents a roller bearing fixed to the roof, and A is the part of the hydraulic cylinder bored to fit the ram that raises the crane. We shall find the longitudinal stresses in CH, CG, CF, EG, GF, neglecting all weights except the 6 tons at 21 ft. and the counter-weight of 1½ ton, at 5 ft. from CG.

$$DG = 21\tfrac{1}{4} \times \frac{6\cdot75}{5\cdot25} = 27\cdot3 \text{ ft.}$$

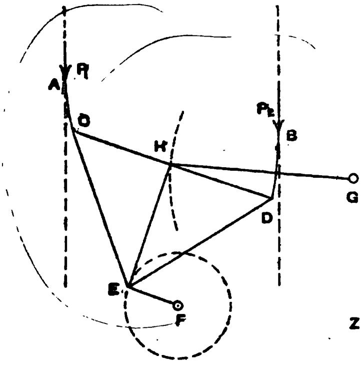


FIG. 18.

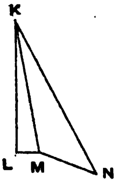


FIG. 19.

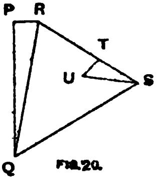


FIG. 20.

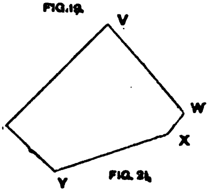


FIG. 21.

The 6 tons may be divided into two parts at D and G, the former being—

$$F_1 = 6 \times \frac{21}{27\cdot3} = 46\cdot1 \text{ tons.}$$

Then, by the triangle of forces, if F_2, F_3 are the stresses in CH and EG,

$$\frac{F_1}{CG} = \frac{F_2}{CD} = \frac{F_3}{DG};$$

$$\therefore F_2 = \frac{4\cdot61 \times 28\cdot1}{6\cdot75} = 19\cdot2 \text{ tons,}$$

$$F_3 = \frac{4\cdot61 \times 27\cdot3}{6\cdot75} = 18\cdot6 \text{ tons.}$$

Similarly, if F_4 and F_5 are the stresses in C F and F G, then the forces at F are proportional to the sides of the triangle C G F, so that

$$F_4 = \frac{1·5 \times CF}{GF} = \frac{1·5 \times 8·4}{6·75} = 1·865 \text{ ton},$$

$$F_5 = \frac{1·5 \times 5}{6·75} = 1·11 \text{ ton}.$$

The vertical components of the stresses in C D and C F are clearly equal to 6 tons and 1½ ton; therefore the stress in C G is 7½ tons.

Fig. 18 represents the outline of the moving parts of Fleming and Ferguson's quadruple-expansion engine; the high-pressure and first intermediate pistons being connected to one crank, as shown, and the second intermediate and low-pressure pistons to another, which is opposite to the first; H G is one part of the air-pump lever. To avoid a confused stress diagram, it is best to proceed as follows:—

P_1 is assumed equal to P_2; of course, in practice their values would be known. The turning force F_1 may be supposed to act perpendicular to E F, and so as to cause equilibrium. We commence with the point A; draw K L equal to P_1, and L M, K M, fig. 19, perpendicular to K L and parallel to A C respectively, so that L M represents the pressure on the slipper, and M K the compressive force on A C. The triangle K M N represents the stresses in C H and C E respectively, of which the former is tensile and the latter compressive. At the points H and E there are three unknown forces; we cannot, therefore, proceed in either direction, but must take the point D, at which P Q, fig. 20, is drawn equal to P_2, and R P, Q R are the pressures on the slipper and the compressive force in B D. Q S and S R are parallels to E D, and D H are the stresses in these members, the former being compressive and the latter tensile. At the point H there are two opposed tensile stresses, R S to the right, N M to the left. Make R T = N M, and draw T U, U S parallels to H E and H G; the stresses in H E and G H are both compressive. Lastly, V W X Y Z represents the polygon of forces at the point E. V W, W X, X Y are first drawn to represent the now known stresses in C E, H E, and D E, and Y Z and Z W, parallel and perpendicular to E F, represent the compressive stress in E F, and the turning force on the crank pin.

Fig. 22 represents a crane in which A B, the jib, is 15 ft. long, and the two stays B C and B C¹ are each 23 ft. long, while $C C^1$ is 6 ft., and $A C$, $A C^1$ are each 10 ft. The weight lifted is 2 tons, the chain being approximately in a line which bisects the angle $C B C^1$. We intend to find the stresses in the jib and in the four parts of the two stays, viz., $B D$, $B D^1$, $D C$, $D^1 C^1$. A counterbalance weight W^1 is supposed to act between $C C^1$, so that the load of 2 tons is properly balanced. By constructing the isosceles triangles $B C C^1$, $A C C^1$ it will be found that the elevation $b c$ of $B C$ is 22·8 ft., and $a c$, that of $A C$, is 9·54 ft. The triangle $a b c$ should next be drawn, fig. 22*a*, and a perpendicular $a d$ drawn to $a c$.

Then the triangle $b d a$ is the triangle of the three forces at the point b. Let F_1 be the resultant of the chain tension

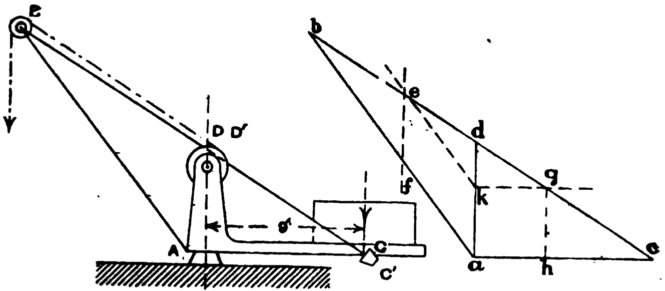


FIG. 22.

FIG. 22 a.

and the stresses in the two ties $B D$, $B D^1$, and let F_2 be the stress on the jib. Then, since $W = 2$ tons,

$$\frac{2}{a d} = \frac{F_1}{b d} = \frac{F_2}{a b}$$

$$F_1 = \frac{2}{4{\cdot}95} \times 12{\cdot}04 = 4{\cdot}86 \text{ tons tensile,}$$

$$F_2 = \frac{2}{4{\cdot}95} \times 15 = 6{\cdot}06 \text{ compressive.}$$

Since the tension of the chain is 2 tons, the resultant tension of the tie rods is

$$F_3 = F_1 - 2$$
$$= 2{\cdot}86.$$

The tension of $B D$, $B D^1$ may now be found in a manner similar to that in which we found the stress on each of the front legs of the sheer legs, figs. 10 and 11. A little thought will also show that if F_4 is the stress in B D,

$$\frac{2\,F_4}{B\,C} = \frac{F_3}{b\,c}.$$

$$F_4 = \frac{2{\cdot}86 \times 23}{2 \times 22{\cdot}8} = 1{\cdot}44 \text{ ton.}$$

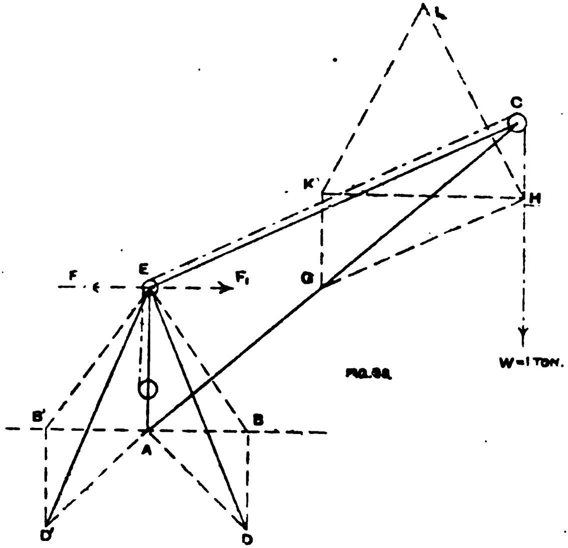


FIG. 9A.

The counterbalance is found by taking moments about the post.

$$W \times 11{\cdot}23 = W^1 \times 9\,;$$

$$\therefore\ W^1 = 2{\cdot}495 \text{ tons.}$$

Let F_5 be the resultant on the back stays; then *d a c* is the triangle of forces for the point *c*.

$$\frac{W^1}{d\,a} = \frac{F_5}{d\,c}$$

$$F_5 = \frac{2{\cdot}495 \times 10{\cdot}72}{4{\cdot}95} = 5{\cdot}4 \text{ tons.}$$

Let F_6 be the stress in each stay; then

$$\frac{2\,F_6}{BC} = \frac{F_5}{bc}$$

F_6 being obtained from F_5 in the same way as F_4 from F_3.

$$\therefore F_6 = \frac{5\cdot4}{2} \times \frac{23}{22\cdot8}\ 2\cdot725 \text{ tons.}$$

The above method is very simple to anyone who can use a slide rule, because arithmetic is always preferable under these circumstance to graphic methods. Failing this, however, take $ak = W$, and draw ke, ef parallels to ab, ad; then

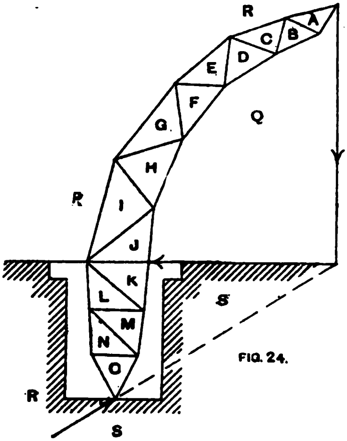


FIG. 24.

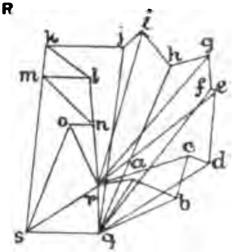


FIG. 25.

$$F_1 = eb$$
$$F_2 = bf.$$

Again, take

$$gh = W^1;$$

then $gc = F_5$.

Fig. 23 represents the outline of a hand crane much used for building purposes. E A, A D^1, A D are equal, and D A D^1 is a right angle. E C, A C are tie rod and jib, which, to make the problem more interesting, are swung round so that their plane is perpendicular to its central position when it bisects the angle between the planes D E A and D^1 E A. The chain is shown by the chain dotted line. There are three forces at E: the pull of the jib and chain along E C, the pull of the post E A and chain downwards, and the resultant of the stresses in D E and E D^1, which is horizontal and in the plane A E C. This last must be horizontal, because the intersection of the planes D E D^1 and C E A is a horizontal line, and three forces, if in equilibrium, must lie in one plane.

Then, if F^1 is the stress of tie E C and chain, and F_2 the stress of the jib A C,

$$\frac{W}{EA} = \frac{F_1}{EC} = \frac{F_2}{CA}$$

and as the stress on the chain is W, that on the tie rod is F − W. F_1 and F_2 may also be found graphically by taking G H parallel to E C and C H equal to W.

Next, if G K, H K be drawn vertical and horizontal, then G H K is the triangle of forces for the point E. The stress in the post is G K − W, and K H is the resultant force of the back stays. Now, since E A, A D, A D^1 are equal, and D^1 A D, E A D^1, E A D are all right angles, the triangle E D D^1 is equilaterial, and as D D^1 is parallel to F, the triangle E D D^1 has its three sides parallel to the three forces at E, viz., the stresses in D E, D E^1, and the force F^1, which is equal and opposite to their resultant F, and which is exerted upon the back stays by the joint at E; therefore D^1 E D is the triangle of forces for the point E, showing that the stress in E D is compressive, that in D^1 E is tensile, and both are equal to F—*i e.*, to H K. Even if the triangle E D D^1 were not isosceles, still the three stresses would be represented by D^1 D, D^1 E, and E D, and a triangle K H L, drawn similar to D^1 D E, would give the required stresses.

An excellent illustration of a case in which we can apply the method of areas to simplify our work is shown in fig. 24. As there are three external forces, they must meet in a point, as shown. The areas may next be lettered, and the diagram of stresses drawn, fig. 25, commencing with the triangle *r s q*, where *r q* = the load. The reader will now have no difficulty in following out the rest of the diagram, if he will find the points *a*, *b*, *c*, &c., by drawing *q a*, *r a* to meet in *a*; *a b*, *q b* to meet in *b*, and so on.

In fig. 26 is shown a centre line sketch of a swivelling crane designed to lift a load not exceeding 15 cwt. Supposing the stresses to be the same as if there were free joints at

A, B, C, D, by the aid of a diagram estimate the stress on B D and on each of the two rods which join B to C. Determine also the greatest straining actions on A C. In fig. 26 let the area between the weight's line of action and C D A be *d*, to the right of the weight and above C B let the

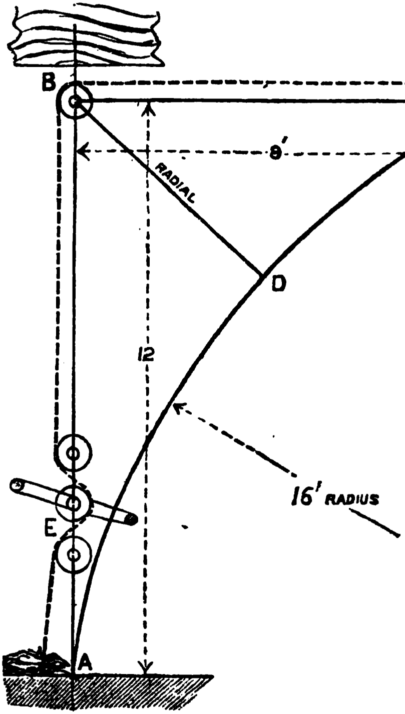


FIG. 26.

area be *e*, from B to A the area is *c*, and let the triangles B D C, B D A, enclosed by straight lines joining C D, D A, be *a* and *b*. The straight lines C D, D A show the directions of the forces at C and A, so that bending and compression act in C D, D A. Let P be the force at C along C D, and let

r be its perpendicular distance from the centre of C D; let A_1 be the section of C D, and Z_c its modulus to resist compression. The stress in C D is

$$\frac{P}{A_1} + \frac{Pr}{Z_c},$$

for the explanation of which see Chapter XI.

Fig. 27 shows the stress diagram; *e a* is the sum of the tensions in the two rods B C, and the pull of the chain $\frac{1}{2}$ W; *b c* is the compression in B E, less the tension of the chain whose pull adds to the compression.

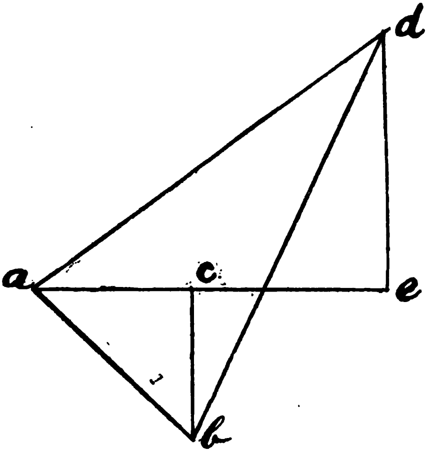


FIG. 27.

Suppose a wheel to be so constructed that the periphery consists of twelve equal pieces, capable of withstanding compression, and jointed together at points which are attached to the central hub by twelve slender rods of equal length, capable of withstanding tension only. Suppose that the wheel is mounted on a central horizontal shaft, which is supported in bearings, and loaded with a weight W at each of the twelve joints, and the wheel to be in such a position that two of the radial rods are vertical, the upper one being free from stress. Draw the stress diagram, showing the stress on each member of the structure, and explain why the tension of the lower vertical radial rod will always be equal to 4 W, however many divisions there are in the wheel periphery, there being a weight W at each point.

In fig. 28 the stress diagram may be drawn by commencing with the triangle of forces at A, and taking B, C, D, &c., in order, remembering the load W at each joint. This the reader will have no difficulty in doing if he has read the previous examples, but to prove that the force in O G is 4 W, whatever the number of sides, is somewhat harder, and may be best shown by trigonometry.

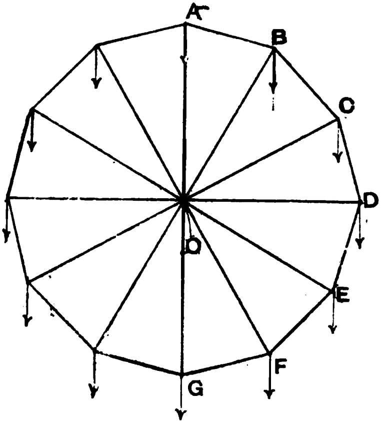


FIG. 28.

Let $2n$ be the number of sides of the structure, then the angle A O B $= \frac{\pi}{n}$, so that the stress in A B is—

$$t_1 = \frac{w}{2 \sin \frac{\pi}{2n}}$$

$$t_2 = t_1 + \frac{w \sin \frac{\pi}{n}}{\cos \frac{\pi}{2n}}$$

where t_2 is the stress in B C.

$$t_3 = t_2 + \frac{w \sin \frac{2\pi}{n}}{\cos \frac{\pi}{2n}}$$

where t_3 is the stress in C D, and so on.

$$t_n = t_{n-1} + \frac{w \sin \frac{n-1}{n}\pi}{\cos \frac{\pi}{2n}}.$$

Let R be the pull in O G.

$$R = w + 2 t_n \sin \frac{\pi}{2n}.$$

$$\therefore R = 2w + 2 \frac{\sin \frac{\pi}{2n}}{\cos \frac{\pi}{2n}} w \Sigma \sin k \frac{\pi}{n},$$

where

$$\Sigma \sin \frac{k\pi}{n}$$

implies the summation of a series of sines in which k varies between 1 and $(n - 1)$. Hence

$$R = 2w + 2w = 4w.$$

CHAPTER II.

BENDING MOMENTS.

It will be as well to mention several important rules in mechanics before commencing this subject. They are as follow :—

(1) The algebraic sum of the moments of any number of forces about a point is equal to the moment of their resultant.

(2) The moment of a couple is the same about any point in its plane.

(3) If any system of forces in a plane acting on a rigid body be in equilibrium, the sum of their moments about any

point is zero; and if each force be resolved into two components parallel to two fixed lines at right angles to one another, the algebraic sum of the components parallel to each line will be zero.

Beams Resting on Two Supports, and Loaded in any Manner Whatever.—Let A B, fig. 29, represent a beam H K, loaded with two uniformly distributed loads, W_1, W_5 of 2 and 6 tons, and let W_2, W_3, and W_4 be concentrated loads of 1, 4, and 5 tons. If we wish to find the bending moments at any point of this beam, it will be necessary to first find F_1 and F_2, the reactions at A and B. We treat A B as a lever, whose fulcrum is at B—or, in other words, take moments about B. Then,

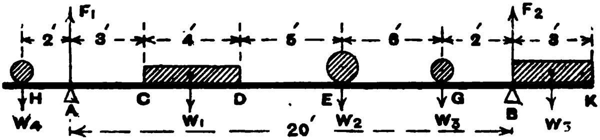


FIG. 29.

$$F_1 \times AB = W_1 \times MB + W_2 \times EB + W_3 \times GB + W_4 \times HB - W_5 \times LB,$$

where L and M are the middle points of B K and C D; and taking the dimensions on fig. 29,

$$F_1 = \frac{2 \times 15 + 8 + 4 \times 2 + 5 \times 22 - 6 \times 1{\cdot}5}{20}$$
$$= 7{\cdot}35 \text{ tons.}$$

F_2 can be found by taking moments about A, or, better, as follows—

$$F_2 = W_1 + W_2 + W_3 + W_4 + W_5 - F_1$$
$$= 18 - 7{\cdot}35 = 10{\cdot}65.$$

We can now find the bending moment at any point of the beam. Let us first take C. Fig. 30 shows the portion of the beam H C, which is kept in equilibrium by W_4 at H, F_1 at A, a vertical shearing stress F_3 exerted by the right-hand part of the beam beyond C upon the left, and a couple N (which we shall call positive if it would cause rotation in the opposite direction to the hands of a clock, and negative if in the same direction). Making use of rule (3) above, we obtain—

$$\begin{aligned} N_1 &= F_1 . AC - W_4 . HC \\ &= 7{\cdot}35 \times 3 - 5 \times 5 \\ &= -\ 2{\cdot}95 \text{ foot-tons,} \end{aligned}$$

showing that its direction is negative and in the opposite direction to that indicated by the arrows P, P_1, in fig. 30.

The bending moment at any point between C D is obtained as follows: Let the point Q, fig. 31, be 2½ ft. to the right of C. The portion H Q is kept in equilibrium by W_4, F_1, five-eighths of W_1, F_4, the shearing stress at Q, and the couple N_2 at Q. Taking moments about Q,

$$N_2 = F_1 \times AQ - W_4 \times HQ - \tfrac{5}{8} W_1 \times \frac{CQ}{2}.$$

The last term in the above is obtained thus: By rule (1) the sum of the moments of the loads distributed on C Q =

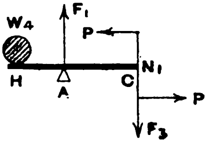


FIG. 30.

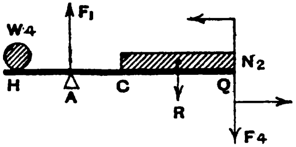


FIG. 31.

the moment of their resultant R, fig. 31, which acts at a distance ½ C Q from Q, and the load on C Q is $\frac{5}{8} W_1$. Therefore its moment about Q is $\frac{5}{8} W_1 \times \frac{CQ}{2}$

$$\begin{aligned} \therefore N_2 &= 7{\cdot}35 \times 5\tfrac{1}{2} - 5 \times 7\tfrac{1}{2} - \tfrac{5}{8} \times 2\tfrac{1}{2} \\ &= 1{\cdot}34 \text{ foot-ton.} \end{aligned}$$

Lastly, let us take a point S midway between W_2 and W_3. The moment obtained will be exactly the same whether we consider the right or left portion of the beam; the portion S K is chosen because there are fewer forces upon it, and therefore less calculation is needed. Taking moments about S, the couple

$$\begin{aligned} N_3 &= W_3 \times SG + W_5 \times SL - F_2 \times BS \\ &= 2{\cdot}25 \text{ foot-tons,} \end{aligned}$$

which is negative, because it would cause rotation in the direction of motion of the hands of a watch.

These three examples will show how to find the bending moment at any point of a beam having two supports, however it may be loaded.

It is sometimes necessary also to find the shearing stress at a point. In fig. 30, since the algebraic sum of the vertical forces must be zero,

$$F_3 = F_1 - W_4 = 7{\cdot}35 - 5 = 2{\cdot}35 \text{ tons.}$$

In fig. 31,

$$F_4 = F_1 - W_4 - R = 7{\cdot}35 - 5 - 1{\cdot}25$$
$$= 1{\cdot}1 \text{ ton.}$$

In fig. 32,

$$F_5 = F_2 - W_3 - W_5 = 10{\cdot}65 - 4 - 6 = {\cdot}65.$$

Beams Fixed at Both Ends.—In this case there are couples at A, B, fig. 33, which compel the ends to take some direction, but unfortunately in practice, although we can frequently find cases in which the beam is fixed at both ends, we cannot tell what angle the ends A, B make with the horizontal. An exceedingly small alteration makes a considerable difference in the bending moment at different sections. Even when free the angle is very small. When the section of the beam is uniform, and the ends are horizontal, and there is a load W at the centre, the bending moment is $\frac{W\,l}{8}$ at both centre and ends, and there is no bending midway between the centre and ends. If the load is uniformly distributed over the beam, the moment at the ends is $\frac{W\,l}{12}$ and at the centre $\frac{W\,l}{24}$ the point of no bending is $\frac{l}{2\sqrt{3}}$ from the centre.

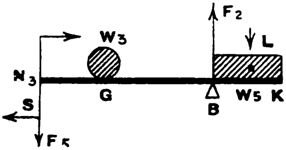


FIG. 32.

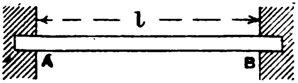


FIG. 33.

A beam or girder must be made sufficiently strong to resist the shearing and bending at every section. Every section has what are called moduli with respect to tension and compression, which are usually represented by the symbols Z_t and Z_c, so that if f_t and f_c are the greatest tensile and compressive stresses produced by the moment M at any section, then

$$M = f_t Z_t = f_c Z_c.$$

We shall now explain how these moduli are obtained. Fig. 34 shows a rectangle whose breadth is b and thickness t, which latter is indefinitely small compared with y, its mean distance from a straight line B C, parallel to its length. The moment of inertia of this area about B C is

$$I = b\,t\,y^2 = A\,y^2,$$

where A is the area of the rectangle. If an area S, fig. 35, be divided into a number of indefinitely thin rectangles,

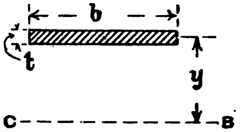


FIG. 34.

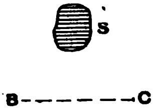


FIG. 35.

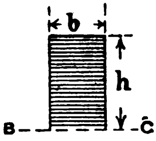


FIG 36.

such as the above, whose distances from B C, breadths, and thicknesses are $y_1\ b_1\ t_1$, $y_2\ b_2\ t_2$, &c., then the moment of inertia of S about B C is

$$\begin{aligned} I &= b_1\,t_1\,y_1{}^2 + b_2\,t_2\,y_2{}^2 + \text{\&c.} \\ &= \Sigma\,A\,y^2 \end{aligned}$$

i.e., equals the sum of the moments of inertia of all the rectangles.

The moment of inertia of the rectangle in fig. 36 is found by the aid of the integral calculus to be

$$\frac{b\,h^3}{3}.$$

Assuming this, the moments of other sections can be readily calculated. The moment of the rectangle in fig. 37 is

$$\frac{B\,(H^3 - h^3)}{3},$$

because it is the difference of the moments of two rectangles. In fig. 38 the moment of each section is

$$\frac{B\,H^3 - b\,h^3}{3}$$

because in (1) there are two rectangles, each of height h and breadth $\frac{b}{2}$, subtracted from the complete rectangle B H, and in (2) there is one rectangle $b\,h$ to subtract.

Now, let a number of forces act on an area such that the forces below $x\,z$ are acting towards us, and those above in the reverse direction, all being perpendicular to the plane of the paper. Then the moments of the forces about $x\,z$ may all be added to one another; and if the sum of the forces above $x\,z$ is equal to the sum of those below, we get a couple.

Next let us suppose that the forces are proportional to the product of the areas on which they act and their distances from $x\,z$. Then, taking a very thin rectangle $b\,t$, whose mean distance from $x\,z$ is y, the total force on it is

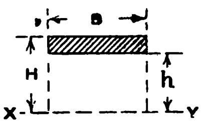


FIG. 37.

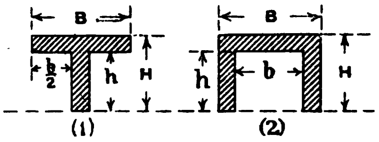


FIG. 38.

$$k\,y\,b\,t,$$

where k is some constant.

Hence the sum of the forces on the upper part

$$= \Sigma\,k\,y\,b\,t,$$

the summation referring to the area above $x\,z$; and if we want a couple, the sum of the forces on the lower part must also

$$= \Sigma\,k\,y\,b\,t.$$

But $\Sigma\,y\,b\,t$ is the moment of the area above $x\,z$ about that line, treating it as if it were a uniform plate having weight; and since the moments of the upper and lower parts are equal, the area would balance about a horizontal axis $x\,z$—in other words, the centre of area lies in $x\,z$.

When a couple acts on a section of a beam or girder it must be balanced by a couple produced by the stresses of the section; hence, assuming at present that the stresses vary as their distances from the line of no stress $x\,z$, it is clear that $x\,z$ must pass through the centre of area of the section, and it is obviously perpendicular to the plane of the couple acting on the section. If we imagine a very long beam of uniform section bent to form a circle, it is subject to bending only, and there is no shearing stress. Let r, fig. 40, be the radius at which there is no stress, the natural length of the beam being $2\pi r$; let r_3 be any other radius less than r, then the compression is

$$2\pi r - 2\pi r_3 = 2\pi y \text{ (fig. 41),}$$

and as stress is proportional to strain, the compressive stress per square inch at that radius is proportional to y, the distance from $x z$. If we suppose this stress to act on a very thin rectangle $a\,b\,c\,d$, whose area is A, the moment of this force about $x z$ is

$$k\,\mathrm{A}\,y^2$$

At a radius r_4 greater than r, the stress is tensile, the increase of length being

$$2\pi(r_4 - r) = 2\pi y_1$$

and the moment of the force on a very thin rectangle whose area is A, is

$$k\,\mathrm{A}\,y_1{}^2$$

Hence the sum of the moments of the forces on the section is

$$\Sigma\,k\,\mathrm{A}\,y^2 = k\,\mathrm{I}$$

I being the moment of inertia of the whole section about $x z$.

Let $f_t\,f_c$ be the greatest tensile and compressive stresses at radii $r_2\,r_1$, and let

$$r_2 - r = h_t$$
$$r - r_1 = h_c$$

then $h_t\,h_c$ are the points on the section furthest from $x z$, and

$$f_t = k\,h_t\,;\ f_c = k\,h_c$$

$$\therefore k = \frac{f_t}{h_t} = \frac{f_c}{h_c}$$

$\therefore$ the moment of resistance of the section to bending is

$$k\,\mathrm{I} = \frac{f_t\,\mathrm{I}}{h_t} = f_t\,\mathrm{Z}_t$$

$$= \frac{f_c\,\mathrm{I}}{h_c} = f_c\,\mathrm{Z}_c$$

Calculation of the Moduli of Unsymmetrical Sections.—It is first necessary to find the centre of area; in fig. 42 we have a double T section with the upper or compression flange smaller than the tension flange, as is usual in cast-iron girders.

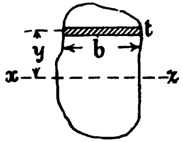
Let G be the centre of area, and $\bar{x}$ its height above A B.

Then, taking moments about A B, and treating the three rectangles formed by flanges and web as if they had weight proportional to their area—

$$\bar{x}(9 + 10 + 3) = 9 \times \tfrac{1}{2} + 10 \times 6 + 3 \times 11\tfrac{1}{2}.$$

$$\bar{x} = \frac{99}{22} = 4\tfrac{1}{2}.$$

FIG. 39.



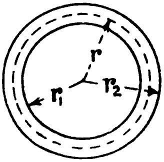


FIG. 40.

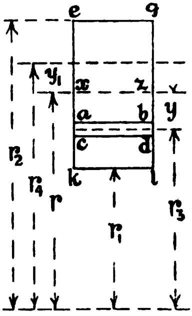


FIG. 41.

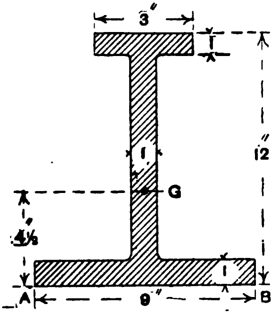


FIG. 42.

In fig. 35 (1) we found that the moment of inertia of the T was

$$\frac{\mathrm{B}\,\mathrm{H}^3 - b\,h^3}{3},$$

so that in this case the moments of the upper and lower are

$$\frac{3 \times (7\tfrac{1}{2})^3 - 2 \times (6\tfrac{1}{2})^3}{3}$$

and
$$\frac{9 \times (4\tfrac{1}{2})^3 - 8 \times (3\tfrac{1}{2})^3}{3}$$

$$\therefore \mathrm{I} \doteq 396,$$

which is the sum of the above.

$$Z_t = \frac{I}{h_t} = \frac{396}{4\cdot5} = 88.$$

$$Z_c = \frac{I}{h_c} = \frac{396}{7\cdot5} = 52\cdot7.$$

This method is far superior to the graphic method of finding the modulus, and is especially easy if a slide rule is used.

A second example is shown in fig. 43, where there are two plates connected by a channel iron. Strictly speaking, we should take into account the rivet holes, but have neglected these for simplicity. The corners and ends of the channel iron are also rounded in actual practice.

Taking moments about A B to find the centre of area,

$$23\bar{x} = 14 \times 7 + 4 \times 12 + 5 \times 13\tfrac{3}{4}.$$

$$\bar{x} = 9\cdot33.$$

$$I = \tfrac{1}{3}[(9\cdot33)^3 + 12 \times (4\cdot67)^3 - (\cdot67)^3 - 10 \times (4\cdot17)^3],$$

$$= 438\cdot4.$$

$$Z_t = \frac{438\cdot4}{9\cdot33} = 47.$$

$$Z_c = \frac{438\cdot4}{4\cdot67} = 94.$$

To Design a Section to Resist a Given Bending Moment.— If the beam is of cast iron, the compression flange is less than the tension flange, because the resistance of cast iron is greater to the former kind of stress than to the latter; the greatest stresses allowed are 2 tons in tension and 4 to 5 tons in compression, although the resistance of cast iron to pressure is about six times its resistance to tension. If the flanges were made to suit this ratio, casting would be difficult. The web should not have a thickness much less than that of the flanges, for the same reason.

Let $x\,z$ be the centre of area of the section, and A_t, A_c, A the areas of the lower and upper flanges and of the web. To simplify the calculation, small areas are omitted, as shown in fig. 44, and we assume that the stresses upon the flanges are $f_t\,f_c$. Then, since we know the bending moment M that the section must resist, we have—

$$M = f_c Z_c = f_t Z_t$$

$$= f_c \frac{I}{h_c} = f_t \frac{I}{h_t}.$$

$$\therefore \frac{f_c}{h_c} = \frac{f_t}{h_t}$$

$$h_c = \frac{f_c h}{f_c + f_t}$$

$$h_t = \frac{f_t}{f_c + f_t} h.$$

Thus we have the position of the centre of the area of the section, and therefore, if we take moments of areas about A B (fig. 44),

$$A\frac{h}{2} + A_c h = (A_t + A_c + A) h_t.$$

$$\therefore A_t = A_c \frac{f_c}{f_t} + A \frac{f_c - f_t}{2 f_t} \quad . \quad . \quad . \quad . \quad (1)$$

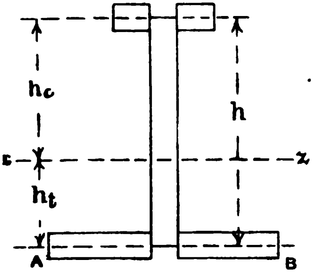


FIG. 44.

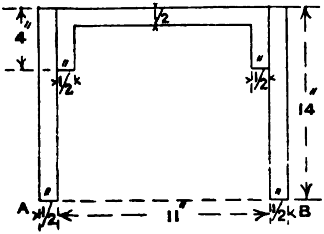


FIG. 43.

$$I = A_t h_t{}^2 + A_c h_c{}^2 + \frac{t(h_t{}^3 + h_c{}^3)}{3},$$

where *t* is the thickness of the web.

$$I = A_t h_t{}^2 A_c h_c{}^2 + A \frac{h_t{}^2 + h_c{}^2 - h_c h}{3}$$

$$= h_t{}^2 \left(A_c \frac{h_c}{h_t} + A \frac{h_c - h_t}{2 h_t} \right) + A_c h_c{}^2 + A \frac{h_t{}^2 + h_c{}^2 - h_c h_t}{3}$$

$$= A_c h h_c - A \frac{h_t{}^2}{6} A h_c \left(\frac{h_c}{3} + \frac{h_t}{6} \right)$$

$$Z_c = \frac{I}{h_c} = h A_c + \frac{A h_t}{6} \left(1 - \frac{h_t}{h_c} \right) + \frac{2 A h_c}{6}$$

$$= h \left\{ A_c + \frac{A}{6} \left(2 - \frac{f_t}{f_c} \right) \right\} . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Also $$Z_t = h \left\{ A_t + \frac{A}{6}\left(2 - \frac{f_c}{f_t}\right) \right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

We obtain the section required in the following way. Z_t, f_t, f_c are known, and h is assumed as 1-10th to 1-15th of the span. A is also assumed from practical considerations. Then equation (2) or (3) gives us A_c, A_t, and equation (1) the remaining area.

CHAPTER III.

NUMERICAL EXAMPLES.

STRESS IN ENGINE GUIDE BAR.

Example I.—What is the greatest stress in the guide bar of an engine? The stroke is 2 ft., and the length of the connecting rod 5·5 ft. The bar is 7 in. broad by 3 in. deep, and the greatest thrust on it may be assumed as taking place at the middle of its span of 3 ft., and when the crank is at right angles to the line of stroke. The pressure on the piston is 9·5 tons.

In fig. 45 the connecting rod is Q C, and the crank S C; F is the thrust of the slide bar on the crosshead. Then

$$\begin{aligned} QS &= \sqrt{QC^2 - CS^2} \\ &= \sqrt{5{\cdot}5^2 - 1} \\ &= 5{\cdot}41. \end{aligned}$$

$$\frac{F}{CS} = \frac{P}{QS}$$

$$F = \frac{9{\cdot}5}{5{\cdot}41} = 1{\cdot}75 \text{ ton.}$$

If b, h are breadth and depth of a rectangular section—

$$Z_c = Z_t = \frac{b\,h^2}{6} = Z.$$

$$\begin{aligned} \therefore f &= \frac{M}{Z} \\ &= \frac{6\,F\,l}{4\,b\,h^2} \text{ where } l = \text{span.} \\ &= \frac{6 \times 1{\cdot}75 \times 36}{4 \times 7 \times 9} \\ &= 1{\cdot}5 \text{ ton.} \end{aligned}$$

The above example is taken from practice. The load of 9½ tons is the pressure upon the H.P. piston of a triple-expansion engine when starting, the guide bar being of mild steel. In these pages we shall, wherever possible, give practical examples. We mention this, as it will probably make them more interesting to the reader.

STRESS IN BOILER GIRDER STAY.

Example II.—What is the stress in the girder stay of the combustion chamber of a marine boiler, fig. 46? The width of the combustion chamber is 28 in., and the stays are 8¼ in. apart from centre to centre, and the pressure per square inch is 150 lb.

FIG. 45.

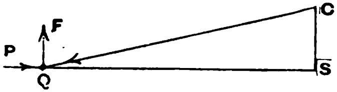
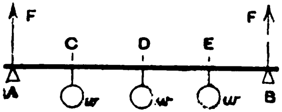


FIG. 47.



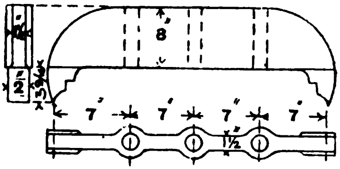


FIG. 46.

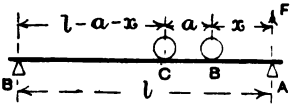


FIG. 48.

The three bolts support three-quarters of the pressure on the top of the combustion chamber, the front and back plates taking the remainder. The problem is therefore reduced to that of a beam, fig. 47, whose span A B is 28 in., and which carries three equal loads w at C, D, E, and

$$w = \frac{150}{4} \times 29 \times 8\tfrac{1}{4} \text{ lb.}$$

The reaction $F = \frac{3}{2} w.$

The moment at D is

$$\begin{aligned} M &= F.BD - w.DE \\ &= \frac{3}{2} w \times 14 - w \times 7. = 14 w. \\ &= 125600 \text{ inch-pounds}; \end{aligned}$$

and
$$f = \frac{M}{Z} = \frac{6\,M}{b\,h^2}$$
$$= \frac{6 \times 125600}{1\frac{1}{2} \times 64}$$
$$= 7850 \text{ lb.}$$

STRESS IN BOILER STAY BY LLOYD'S RULE.

Example III.—What stress is allowed by Lloyd's in girders such as the above? Lloyd's rule being as follows:—

$$\frac{C \times d^2 \times T}{(L - P)\,D \times L} = \text{working pressure} = p.$$

L = length of girder in inches.
P = pitch of stay in inches.
D = distance apart of girders in inches.
d = depth of girder at centre in inches.
T = thickness of girder at centre in inches.
C = 6,000, if there is one stay to each girder.
C = 9,000, if there are two or three stays.
C = 10,200, if there are four stays.

We shall assume that
$$L = (n + 1)\,P$$
where L = the number of bolts, and that the width of the combustion chamber is also L. Then the stays will support between them a load
$$n\,w = (L - P)\,D\,p = n\,.\,P\,.\,D\,.\,p.$$
First, take the case of two stay bolts; then at any point between these the bending moment is constant; the reactions at the points of support are each
$$F = \frac{n\,w}{2} = \frac{n\,P\,.\,D\,.\,p.}{2}$$
in this case
$$= P\,.\,D\,.\,p$$
$$\therefore \text{ the bending moment} = p\,D\,P^2$$
and
$$f = \frac{M}{Z} = \frac{6\,p\,D\,P^2}{T\,.\,d^2\,.}$$
but
$$C = \frac{(L - P)\,D\,.\,L\,.\,p}{d^2\,T}$$
$$= \frac{6\,P^2\,D\,p}{d^2\,T} \;\therefore\; L = 3\,P$$
$$\therefore C = f = 9000 \text{ lb.}$$

If there are three bolts, we shall find that the greatest bending moment, which is at the centre, is

$$M = p \cdot \frac{L - P}{2} \cdot D \cdot \frac{L}{2} - p \frac{L - P}{3} D \frac{L}{4}.$$

$$= \frac{p(L - P)DL}{6} = 2pP^2D \therefore L = 4P.$$

$$\therefore f = \frac{M}{Z} = \frac{12pP^2D}{d^2T},$$

but $$C = \frac{12pP^2D}{d^2T}$$

$$\therefore C = f = 9000\,\text{lb}.$$

If there is one bolt—

$$M = \frac{p(L - P)D.L}{4}$$

$$\therefore f = \frac{3p(L - P)DL}{2d^2T}$$

$$\therefore C = \tfrac{2}{3}f \quad \therefore f = 9000.$$

Lastly, if there are four bolts—

$$M = \frac{3}{20}p(L - P)D.L$$

$$\therefore f = \frac{M}{Z} = \frac{9}{10}\,\frac{p(L - P)DL}{d^2T}$$

$$\therefore C = \frac{10}{9}f.$$

$$f = 9180\,\text{lb}.$$

If the girders are of steel, the above values of C may be increased 10 per cent.

The Board of Trade rules give exactly the same results. The formula in this case is

$$\frac{Cd^2T}{(W - P)DL} = p,$$

the values of C being 500, 750, and 850, because L is in feet. W is the width of the combustion chamber in inches, and the other three quantities are unaltered.

MAXIMUM STRESS IN TRAVELLING CRANE GIRDER.

Example IV.—In a travelling crane of l feet span, the load is supported on a carriage which runs upon two similar girders, the axles of the carriage being a feet apart, and a load of W tons coming upon each wheel. Find the two points where the bending moment is a maximum, and the greatest bending moment at x feet from one end, x being less than half the span.

Firstly, it can be shown by a rather lengthy proof that to obtain the greatest bending moment at any given point on the right half of the girder the right wheel must be upon that point, and for any given point on the left half the left wheel must be over the point. Let us suppose this proved, and let F be the reaction at A. Then

$$\mathrm{F} = \frac{\mathrm{W}\,(l - a - x + l - x)}{l}$$

$$= \frac{\mathrm{W}\,(2l - 2x - a)}{l}$$

$$\therefore \text{ bending moment at B} = \mathrm{F}x$$

$$= \frac{\mathrm{W}x\,(2l - 2x - a)}{l}$$

$$= \frac{\mathrm{W}\,(2lx - 2x^2 - ax)}{l} \text{ foot-tons.}$$

Now, suppose we have a function of x—*i.e.*, a quantity that varies when x varies—and suppose it can be thrown into the form

$$\mathrm{A} + \mathrm{B}x + \mathrm{C}x^2 + \mathrm{D}x^3 + \mathrm{E}x^4 + \text{\&c.}$$

Then this has either a maximum or minimum value, when

$$\mathrm{B} + 2\mathrm{C}x + 3\mathrm{D}x^2 + 4\mathrm{E}x^3 + \text{\&c.} = 0,$$

and common sense will generally tell us whether the function has a maximum or a minimum value, although we can also find this mathematically. If the reader remembers the above fact, he will reap one of the greatest benefits of a study of the differential calculus without having the trouble of mastering a considerable amount of mathematics that is rarely of use to an engineer.

Here, then, the bending moment is greatest when

$$2lx - 2x^2 - ax.$$

is a maximum, or when

$$2l - 4x - a = 0$$

$$\therefore x = \frac{l}{2} - \frac{a}{4}$$

so that the right or left wheel must be $\frac{a}{4}$ feet from the centre of span to cause the greatest moment, which is

$$\frac{W(2l - a)^2}{8l} \text{ foot-tons.}$$

The ram of the fixed hydraulic riveter, fig. 48*a*, is 6 in. diameter, and the accumulator is loaded to 1,500 lb. per square inch. Allow for the momentary increase of pressure, and determine, by an approximate method, suitable dimensions for the transverse section of the cast-steel hob through K K. Determine also a suitable diameter for the two tie bolts, which are of forged steel.

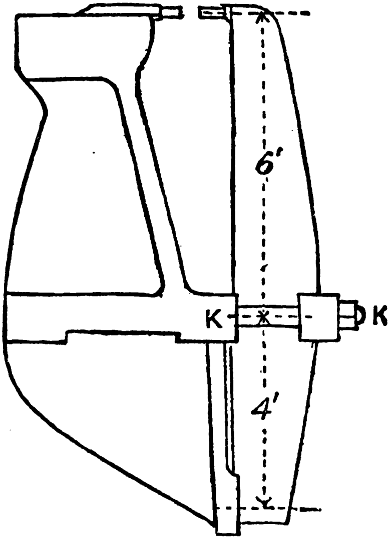


FIG. 48A.

We may assume the momentary increase of pressure to be 2,000 lb. per square inch, so that the bending moment at the section kk of the steel hob is

$$M = 2000 \times \cdot 7854 \times 36 \times 72 \text{ inch-pounds.}$$

The stress that we might assume for cast steel is 16,000 lb. per square inch, the load being entirely removed after each application, if we take the figures given in Unwin's "Elements of Machine Design," Part I., p. 43. But as the load is applied suddenly, it will produce twice the stress calculated from the formula,

$$f = \frac{M}{Z_1}$$

where Z_1 is the modulus of the section at kk. Hence, take $f = 8{,}000$, and supposing the section rectangular, and neglecting the effect of the sides parallel to the plane of the paper; supposing also that 20 in. is the depth of the section parallel to kk between the centres of the areas of the sections of the metal; then

$$bt = \frac{2000 \times \cdot 7854 \times 36 \times 72}{8000 \times 20}$$

$$= 25 \cdot 4 \text{ square inches}$$

where b is the breadth of the section perpendicular to the paper, and t is the thickness of the metal. If the breadth is 12 in., the thickness will be a little over 2 in. The load on the two bolts is

$$P = 2000 \times \cdot 7854 \times 36 \times \frac{10}{4} \text{ lb.}$$

Assuming a stress of 5,000 lb., we have

$$\delta^2 = 18$$

where δ is the diameter at the bottom of the thread.

$$\delta = 4 \cdot 2 \text{ in.}$$

hence the diameter at the top is

$$d = \frac{\delta + \cdot 05}{\cdot 9} = \frac{4 \cdot 25}{\cdot 9}$$

$$= 4 \cdot 72 \text{ in.}$$

CHAPTER IV.

ON TENSILE SHEARING AND COMPRESSIVE STRESSES: RIVETED JOINTS.

IF strength is the only consideration in the design of a joint, then there should be no weakest section. One of the nearest approximations to such a case is that of a riveted joint, although even here the pitch is often smaller than mere strength would require in order to prevent what would otherwise be a leaky joint, and the rivets are more capable of resisting shearing than the plate to resist the tensile stress upon it. The subject of simple forms of riveted joints has been sufficiently dealt with in many text-books. We propose here to treat of some more complicated forms.

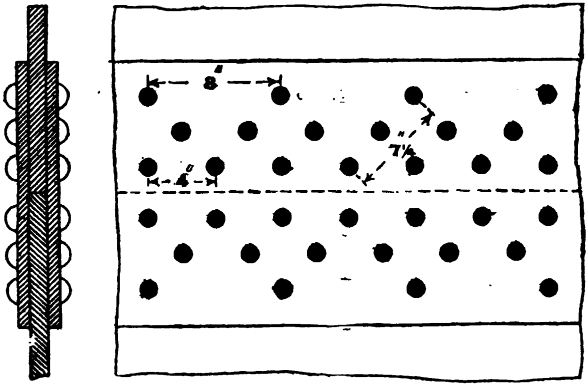


FIG. 49.

First let us consider how such a joint as that shown in fig. 49 might give way. This is a treble-riveted butt joint, with double straps, and five rivets to one wide pitch of 8 in., suitable for the longitudinal joint of a marine boiler.

Thickness of plate = $1\frac{1}{8}$ = 1·125 in. = t
Rivet diameter d = $1\frac{1}{4}$ = 1·25; area = 1·227 = a
Wide pitch = 8 in.; narrow pitch = 4 in.
Steel plates, steel rivets.

The joint might give way (1) at the wide pitch, (2) by shearing of all the rivets, (3) by shearing of the outer row of rivets at the same time that the plate gave way at the line of rivets next to the wide pitch.

It is usual to compare the resistance in each of these cases with the resistance of the solid plate, and to call this ratio the efficiency.

Let f_s = the shearing stress of a rivet
f_t = the tensile stress of the plate
p = wide pitch
p^1 = narrow pitch

Then the Board of Trade consider that

$$\frac{f_s}{f_t} = \frac{23}{28}$$

and that a rivet in double shear is 1·75 as strong as a rivet in single shear.

The efficiency at the wide pitch is

$$\frac{p - d}{p} = \cdot 84.$$

There are five rivets to one wide pitch of 8 in., so that their resistance to shearing is

$$1\cdot75 \times 5 f_s a,$$

and the resistance of the undrilled plate for a pitch of 8 in. is $f_t\,p\,t$. Hence the efficiency for the rivet section is

$$\frac{1\cdot75 \times 5 f_s a}{f_t\,p\,t}$$

$$= 1\cdot75 \times 5 \times \frac{23}{28} \times \frac{1\cdot227}{1\cdot125 \times 8} = \cdot98.$$

In the third possible case of failure the resistance of the outer row of rivets for each wide pitch is $1\cdot75\, f_s a$, since there is only one rivet in double shear; and the resistance of the plate at the outer narrow pitch is $2 f_t (p^1 - d)\, t$, taking two pitches of 4 in. The combined efficiency is

$$\frac{1\cdot75\, f_s a + 2 f_t (p^1 - d)\, t}{f_t\,p\,t}$$

$$= \frac{1\cdot75 \times \frac{23}{28} \times 1\cdot227 + 2 \times (4 - 1\cdot25) \times 1\cdot125}{8 \times 1\cdot125}$$

$$= \cdot883.$$

The joint might also give way at the butt strap, and experiments have shown that it is preferable to make each strap 1¼ times the thickness that theory would dictate, considering its area at the weakest section, which is at the inner narrow pitch of rivets. Let t_1 be the thickness of each strap, then the resistance of both straps at that pitch is $4 f_t\, t\, (p^1 - d)$ for 8 in. of plate, and this must be 1¼ times as strong as the plate at the wide pitch where its efficiency is least.

$$\therefore 4 f_t\, t_1\, (p^1 - d) = \frac{5}{4} f_t\, t\, (p - d)$$

$$\therefore t_1 = \tfrac{5}{16}\, t\, \frac{(p - d)}{(p^1 - d)} = {\cdot}8629 = \tfrac{7}{8} \text{ bare.}$$

Suppose, now, we had to design a similar joint for a marine boiler, 12 ft. 6 in. diameter, pressure 160 lb. per square inch, we should arrange that the efficiency of the plate at the wide pitch is equal to that of the rivets, so that the pitch will not be too small, nor the rivets too large.

It may be well here to remark that the Board of Trade limits the widest pitch to 8½ in., unless specially allowed by them after the case has been submitted to and passed by them.

Let D, P, η be the diameter of the boiler in inches, the pressure per square inch, and the least efficiency of the joint; then

$$\text{P D} = 2 f_t\, \eta\, t$$

where f_t is the safe stress in the plate;

$$\eta = \frac{p - d}{p} \quad \therefore d = p\,(1 - \eta),$$

and for equal strength of rivets and plate

$$(p - d)\, t = 1{\cdot}75 \times 5 \times \frac{23}{28} \times \frac{\pi d^2}{4},$$

because there are five rivets in double shear to one wide pitch, and

$$\frac{f_s}{f_t} = \frac{23}{28}$$

$$\therefore \frac{\eta\, t}{1 - \eta} = 1{\cdot}75 \times \frac{23}{28} \times {\cdot}7854\, d$$

$$= 5{\cdot}64\, d \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (a)$$

$$\therefore d = \frac{\mathrm{P\,D}}{11{\cdot}28\, f_t\,(1-\eta)}$$

and $$f_t = \frac{62720}{5}\ \text{lb.}$$

i.e., 28 tons is the breaking stress, and 5 is the factor of safety.

$$\therefore p = \frac{d}{1-\eta} = \frac{\mathrm{P\,D}}{11{\cdot}28\, f_t\,(1-\eta)^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

Suppose we assume a high efficiency of joint, say 85½ per cent. Then

$$\begin{aligned} p &= \frac{160 \times 150 \times 5}{11{\cdot}28 \times 62720 \times ({\cdot}145)^2} \\ &= 8{\cdot}07\ \text{in.} \\ t &= \frac{\mathrm{P\,D}}{2\, f_t\, \eta} = \frac{160 \times 150 \times 5}{2 \times 62720 \times {\cdot}855} \\ &= 1{\cdot}12 \\ &= 1\tfrac{1}{8}\ \text{bare} \\ d &= p\,(1-\eta) = 8{\cdot}07 \times {\cdot}145 = 1{\cdot}17 \end{aligned}$$

The nearest 32nd of an inch above this is $1\frac{3}{16}$ in., so that if

$$p = 8\tfrac{1}{16},\ t = 1\tfrac{1}{8},\ d = 1\tfrac{3}{16},$$

the plate and rivet efficiencies will be very nearly equal, and the efficiency of the plate section at the outer narrow pitch and of the outer row of rivets is

$$\begin{aligned} &\frac{2\, f_t\,(p^1 - d)\,t + 1{\cdot}75\, f_s\, a}{f_t\, p\, t} \\ &= \frac{2 \times 2{\cdot}844 \times 1{\cdot}125 + 1{\cdot}75 \times \dfrac{23}{28} \times 1{\cdot}107}{8{\cdot}0625 \times 1{\cdot}125} \\ &= {\cdot}88, \end{aligned}$$

so that the least efficiency is ·855.

The butt strap has a thickness t_1, and

$$\begin{aligned} t_1 &= \tfrac{5}{16}\, t\, \frac{p-d}{p^1-d} \\ &= \tfrac{5}{16} \times 1{\cdot}125 = \frac{6{\cdot}875}{2{\cdot}844} \times {\cdot}85\ \text{in.} \\ &= \tfrac{7}{8}\ \text{in. bare.} \end{aligned}$$

Equation (b) shows that the above method of calculation will make p greater than $8\frac{1}{2}$ in. if too high a value of η is assumed; but from equation (a), if t is not greater than d, then

$$\frac{\eta}{1-\eta} \text{ is not less than } 5{\cdot}64,$$

$$6{\cdot}64\,\eta \text{ is not less than } 5{\cdot}64,$$

or

$$\eta \text{ not less than } {\cdot}8495.$$

The Board of Trade require that d shall not be less than t, and therefore η cannot be less than ·8495, if plate and rivet efficiencies are to be equal; so that if the assumption of this efficiency makes p in equation (b) more than $8\frac{1}{2}$ in., then the pitch must be assumed to be $8\frac{1}{2}$ in., and d must be made equal to t. $\frac{\mathrm{P\,D}}{f_t}$ cannot be greater than 2·17 if the efficiencies are equal; for if p and η have their limiting values—viz., $8\frac{1}{2}$ and ·8495—then

$$\frac{\mathrm{P\,D}}{f_t} = 11{\cdot}28\,p\,(1-\eta)^2 = 2{\cdot}17.$$

Suppose, then, that this value is exeeeded, we proceed as follow:—

$$\frac{\mathrm{P\,D}}{2\,f_t} = \eta\,t$$

$$= \frac{p-d}{p}\,t$$

$$= \frac{p-t}{p}\,t$$

and

$$p = 8\tfrac{1}{2} \text{ in.}$$

*Let

$$\mathrm{D} = 15 \text{ ft. } 2\tfrac{7}{8} \text{ in.} = 182\tfrac{7}{8} \text{ in.}$$

$$\mathrm{P} = 170 \text{ lb.}$$

and

$$f_t = 13250 \text{ lb.}$$

$$\frac{\mathrm{P\,D}}{2\,f_t} = 1{\cdot}17$$

$$\therefore 1{\cdot}17 = \frac{(8{\cdot}5 - t)}{8{\cdot}5}\,t$$

$$\therefore \quad t^2 = 8{\cdot}5\,t + 9{\cdot}95 = 0;$$

a quadratic giving

$$t = \frac{8{\cdot}5 \pm 5{\cdot}7}{2}.$$

---

* See the *Engineer*, July 10th, 1891.

Of course we must take the minus sign for the second term, which gives—

$$t = 1{\cdot}4 = 1\tfrac{13}{32} \text{ in., nearly.}$$

$$d = 1\tfrac{13}{32} \text{ in.} ; \; p^1 = 4\tfrac{1}{4} \text{ in.}$$

$$\text{Plate efficiency} = {\cdot}834.$$

$$\text{Rivet efficiency} = {\cdot}93.$$

An example of a pitch greater than $8\frac{1}{2}$ in. is as follows:—

$$D = 15 \text{ ft. } 1 \text{ in.} ; \; p = 8\tfrac{7}{8} \text{ in.}$$

$$P = 160 \text{ lb.} ; \qquad t = d = 1\tfrac{11}{32}$$

$$p^1 = 4\tfrac{7}{16} \text{ (steamship Ophir),}$$

which may be obtained in the same way as in the last example, assuming $p = 8\frac{7}{8}$ and $f_t = 12768$ lb.; or by the first method, assuming an efficiency of ·8495, so that d and t may be equal.

Thus

$$p = \frac{\text{P D}}{11{\cdot}28 f_t (1 - \eta)^2}$$

$$= \frac{160 \times 181}{11{\cdot}28 \times 12768 \times ({\cdot}1505)^2}$$

$$= 8\tfrac{7}{8} \text{ in.}$$

$$d = p(1 - \eta) = 8{\cdot}875 \times {\cdot}1505$$

$$= 1{\cdot}335 = 1\tfrac{11}{32} \text{ in., very nearly}$$

$$\frac{\eta t}{1 - \eta} = 5{\cdot}64\, d$$

$$\therefore t = d = 1\tfrac{11}{32} \text{ in.}$$

The stress f_t here chosen is $28\frac{1}{2}$ tons per square inch breaking stress, with a factor of safety of 5.

There is another type of joint much used for the same purpose as the above, from which it differs only in having the pitch of the inner row of rivets equal to that of the outer row, while the middle line has a small pitch, as usual. There are, therefore, only four rivets to one large pitch, instead of five, and, for a given thickness of plate and pitch, the rivets must be larger than in the first joint, and therefore the efficiency is a trifle less; or if t and η are fixed, p must be greater than with five rivets.

The butt straps are stronger at the inner wide pitch, and therefore need only be five-eighths of the plate, for the weak section at the narrow pitch is supported by the inner line of rivets, and a break at the narrow pitch must be accompanied by the shearing of these to cause a failure of the joint. The following is a numerical example: To design a joint for 1¼ in. steel plates, having an efficiency of ·8325. For equal strength of rivets and plate

$$(p - d)\, t = 1{\cdot}75 \times \frac{23}{28} \times 4\, a\,;$$

because there are four rivets in double shear, and

$$\frac{(p - d)}{p} = {\cdot}8325$$

$$\therefore {\cdot}8325 \times 1\tfrac{1}{4}\, p = 1{\cdot}75 \times \frac{23}{28} \times 4\, a$$

and

$$d = p\,(1 - \eta) = p \times {\cdot}1675$$

$$\therefore d^2 = p^2 \times {\cdot}0281$$

$$\therefore p = \frac{{\cdot}8325 \times 1\tfrac{1}{4} \times 28}{1{\cdot}75 \times 23 \times \pi \times 0281} = 8{\cdot}2 \text{ in.}$$

$$\therefore d = 1{\cdot}375 = 1\tfrac{3}{8} \text{ in.}$$

The straps will be each

$$\frac{5}{8}\, t = \frac{25}{32} \text{ in.}$$

Two lengths of mild steel tie rod of rectangular section 7 in. by 1 in. are required to be connected by means of a riveted butt joint, with a cover plate on each side. Calculate the diameter of the rivets, and estimate the efficiency of the joint.

We may assume eight rivets, four on each side of the joint; there is one nearest the joint, two further from the joint on a line perpendicular to the centre line, and one beyond these, so that the efficiency of the strap is the same as that of the plate. The plate efficiency is clearly

$$\eta_1 = \frac{7 - d}{7}$$

where d is the diameter of the rivet. Assuming

$$f_s = \frac{23}{28} f_t$$

and that a rivet in double shear is 1¾ times as strong as one in single shear,

$$4 \times {\cdot}7854 \times 1{\cdot}75 \times \frac{23}{28} d^2 = (7 - d)\,.\,t\,.$$

$$\therefore\ 4\cdot41\,d^2 + d - 7 = 0.$$

$$\therefore\ d = 1\cdot15,$$

$$\eta_1 = \cdot836\,. = \eta_2$$

where η_2 is the efficiency of the rivets.

η_3 is the efficiency of the joint, supposing that it may fail by breaking at the narrow pitch, and by shearing of the outer rivet.

$$\eta_3 = \frac{7 - 2d}{7} + \frac{1\cdot75 \times \cdot7854\,d^2 \times \frac{23}{28}}{7}$$

$$= \frac{4\cdot7 + 1\cdot9}{7} = \cdot942.$$

The thickness t_1 of the butt strap $= \frac{5}{8}$ that of the plate, because its efficiency is the same as that of the plate.

In figure 50 is shown a form of joint which is a sort of combination of the butt strap and lap joint. The outer pitches a and c are twice as great as the inner pitch b. Let p be the outer pitch, d the rivet diameter, and t the thickness of the plate; we shall show the various ways in which the joint may fail, and calculate the pitch and diameter of the rivets.

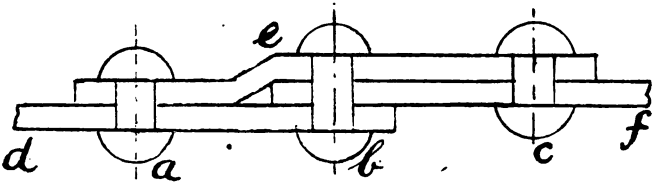


FIG. 50.

The plate may give way along the line of rivets a, and the efficiency at this line is

$$\eta_1 = \frac{p - d}{p}$$

The rivets are weakest at a and b, both of them being in single shear if the plate d is strong enough to pull them from e and f. There are thus three rivets to each wide pitch, so that

$$3 \times \frac{23}{28} \times \cdot7854\,d^2 = (p - d)\,.\,t.$$

The plate *f* might break at the middle line of rivets, and the row *c* of rivets might shear simultaneously ; so that

$$(p - 2d)t + \frac{23}{28} \times \cdot 7854 \times d^2 = (p - d).t.$$

Multiplying the latter equation by 3, and subtracting the former from the latter, we get

$$3(p - 2d)t = 2(p - d).t.$$

$$\therefore p = 4d.$$

$$\eta_1 = \frac{p - d}{p} = \tfrac{3}{4},$$

which will be the efficiency for all three ways of fracture. The first equation also gives us

$$\frac{23}{28} \times \cdot 7854\, d = t$$

$$d = 1 \cdot 55\, t.$$

CHAPTER V.

Cottered Joints.

Another form of joint, in which one would naturally expect that some ratio would exist between the tensile and shearing stresses to which it is exposed, is that shown in fig. 51. It is much used to connect piston rods to crossheads. Let *b*, *t* be the mean breadth and thickness of the cotter. The rod is of wrought iron or steel, the cotter of steel, and the socket of cast iron, wrought iron, or cast steel. Usually P and D have the same diameter, because the rod when in compression is subject to a bending as well as a longitudinal stress, and it is therefore unnecessary to increase the section near the cotter so that it may equal that at the rod ; also an enlarged end requires a split gland for the stuffing box. Let F be the greatest force upon this joint, and let D_2 D_3 be the diameters of the tapered end at the sections *x y*, *w z*, which are taken through the centres of the circles at the ends of the cotter section. The tapered end is weakest at *w z* and the socket at *x y*. The section of the cotter opposed to shearing differs but little from 2 *b t*.

$$\therefore F = 2 f_s b t = f_t \left(\frac{\pi}{4} D^2{}_3 - D_3 t \right)$$

$$= f_t{}^1 \left\{ \frac{\pi}{4} (D_1{}^2 - D_2{}^2) - (D_1 - D_2) t \right\}$$

where f_t, $f_t{}^1$ are the greatest tensile stresses on the rod end and socket at $w z$ and $x y$. When F compresses the rod, there is, of course, no stress on the cotter, but there is a bursting stress on the socket, which, we believe, it would be exceedingly difficult to calculate. The values of f_s, f_t, and $f_t{}^1$, even for engines of the same class, differ very greatly, as will be seen by Table I. It is probable that they are rarely calculated, for, if they are so, it is evident that designers must have widely different views upon the strengths of materials. In Table I., *c*, *m*, *w*, *k* stand for cast iron, malleable cast iron, wrought iron, cast steel. Piston rods and cotters are all made of steel. The letters C, N stand for condensing and non-condensing. In the former case 12 lb. is added to the boiler pressure, so as to allow 3 lb. for back pressure. In the two compound engines the stresses have been calculated from the diagrams. H S stands for horizontal stationary; L for locomotive; P for portable; H S T for horizontal stationary tandem; and R M for rail mill engines. It is sometimes stated in text-books that L = 3 *b*, but in the table it is generally between 2 and 3 *b*. The greatest values

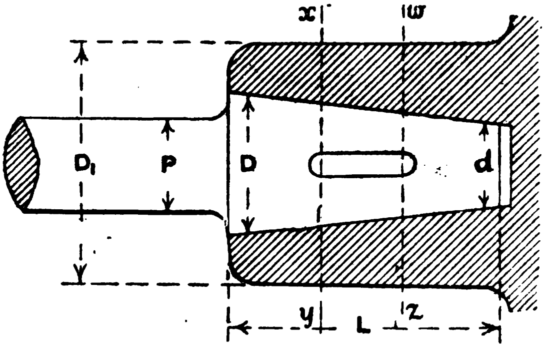


FIG. 51.

of f_s, f_t, and $f_t{}^1$ for stationary engines are 7,840, 11,200, and 1,975; and for locomotives 15,500, 14,500, and 4,010. We have in a few cases calculated f_b, the pressure per square inch on

TABLE I.

Type of engine..	H.S.	H.S.	H.S.	H.S.	H.S.T.	R.M.T.	L.	L.	L.	P.
H.P. diameter ..	9	$14\frac{1}{2}$	27	13	24	34	18	17	17	10
L.P. diameter ..	..	..	..	$22\frac{1}{2}$	46	60	..	..	..	..
Stroke..........	18	30	36	24	72	60	24	24	24	12
Boiler pressure..	60 N	50 C	60 C	120	90	100	150	150	150	80 N
Max. pressure H.P.C.	..	..	..	67	78	..	..	..	..	..
Max. pressure L.P.C.	..	..	..	43	16	..	..	..	..	..
L	$4\frac{1}{4}$	$4\frac{1}{2}$	9	$5\frac{1}{4}$	$14\frac{1}{4}$	13	$5\frac{5}{8}$	$5\frac{1}{2}$	$5\frac{3}{4}$	3
b	$1\frac{3}{8}$	2	$4\frac{1}{8}$	$2\frac{7}{16}$	$5\frac{1}{2}$	$4\frac{7}{8}$	$2\frac{1}{4}$	$2\frac{1}{8}$	$2\frac{1}{4}$	$1\frac{1}{4}$
t	$\frac{3}{8}$	$\frac{1}{2}$	1	$\frac{9}{16}$	$1\frac{1}{4}$	$1\frac{1}{2}$	$\frac{7}{8}$	$\frac{5}{8}$	$\frac{5}{8}$	$\frac{1}{4}$
D_1	$3\frac{1}{2}$ *c*	$4\frac{1}{4}$ *m*	$6\frac{1}{2}$ *w*	4 *w*	$8\frac{1}{2}$ *w*	$9\frac{1}{2}$ *w*	$6\frac{1}{4}$ *k*	..	$4\frac{3}{8}$ *w*	..
D	$1\frac{1}{2}$	$2\frac{3}{8}$	$3\frac{3}{8}$	$2\frac{1}{2}$	$5\frac{1}{4}$	6	$3\frac{3}{8}$	$2\frac{3}{4}$	$2\frac{7}{16}$	$1\frac{5}{8}$
d	$1\frac{1}{4}$	$1\frac{7}{8}$	$2\frac{5}{8}$	$2\frac{1}{8}$	5	5	$3\frac{1}{16}$	$2\frac{1}{2}$	$2\frac{1}{16}$	$1\frac{3}{8}$
P	$1\frac{1}{2}$	$2\frac{3}{8}$	$3\frac{1}{2}$	$2\frac{1}{2}$	$5\frac{1}{4}$	6	$2\frac{3}{8}$	$2\frac{3}{4}$	$2\frac{3}{4}$	$1\frac{5}{8}$
f_s	3700	7600	5000	6950	4500	..	9700	16400	15500	10000
f_t	4110	6670	10750	6330	4360	..	8860	9100	14500	4510
$f_t{}^1$	523	1620	1880	1620	1975	..	2490	..	4010	..
f_b	7525	..	14100	17600	..	..	14000	..	25200	..

the bearing surface of the cotter. This is sometimes greatest in the rod end and sometimes in the socket.

To calculate the dimensions of such a joint as this, we should first assume some ratio—

$$\frac{t}{b} = k = \frac{1}{4} \text{ generally.}$$

Then
$$t^2 = \frac{k\,F}{2 f_s},$$

so that t and b are known.

Then

$$\frac{\pi}{4}D_3^2 - D_3\,t = \frac{F}{f_t}$$

is a quadratic, from which D_3 may be found. If D equals D_1, and L is assumed at from $2\frac{1}{4}b$ to $3b$, this gives the

TABLE II.—DIMENSIONS OF LOCOMOTIVE CROSSHEADS.

Diameter of cylinder..	19	17	16	16	15
Stroke	26	24	24	22	22
Pressure	150	150	150	150	150
A	3	$2\frac{3}{4}$	$2\frac{5}{8}$	$2\frac{1}{2}$	$2\frac{5}{8}$
B	$2\frac{7}{8}$	$2\frac{1}{16}$	$2\frac{7}{16}$	$2\frac{5}{8}$	$2\frac{1}{4}$
C	$2\frac{1}{2}$	$2\frac{1}{8}$	$2\frac{1}{16}$	2	$1\frac{7}{8}$
D	$\frac{11}{16}$	$\frac{5}{8}$	$\frac{5}{8}$	$\frac{5}{8}$	[illegible]
E	$5\frac{1}{4}$	$4\frac{5}{8}$	$4\frac{1}{4}$	$4\frac{1}{4}$	$4\frac{1}{8}$
F	$2\frac{3}{4}$	$2\frac{1}{4}$	$2\frac{1}{4}$	$2\frac{1}{8}$	2
G	$1\frac{5}{8}$	$1\frac{5}{8}$	$1\frac{5}{8}$	$1\frac{5}{8}$	$1\frac{1}{4}$
H	3	3	3	$2\frac{5}{8}$	$2\frac{1}{2}$
I	3	3	3	3	$2\frac{5}{8}$
J	6	$5\frac{3}{4}$	6	$5\frac{5}{8}$	5
K	$2\frac{3}{4}$	$2\frac{1}{2}$	$2\frac{1}{2}$	$2\frac{5}{8}$	$2\frac{1}{4}$
L	$3\frac{1}{4}$	3	3	$2\frac{7}{8}$	$2\frac{3}{4}$
M	$1\frac{5}{8}$	$1\frac{5}{8}$	$1\frac{5}{8}$	$1\frac{3}{4}$	$1\frac{1}{2}$
N	14	10	10	$10\frac{1}{2}$	9
O	$11\frac{1}{4}$	$11\frac{1}{4}$	$10\frac{3}{4}$	$10\frac{1}{4}$	10
P	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{9}{16}$
Q	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
R	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{9}{16}$
S	$6\frac{1}{2}$	$6\frac{1}{2}$	6	$5\frac{1}{4}$	$6\frac{1}{8}$
T	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
U	$2\frac{7}{8}$	$2\frac{3}{4}$	$2\frac{3}{4}$	$2\frac{1}{2}$	$2\frac{1}{2}$
V	$1\frac{7}{8}$	$1\frac{3}{4}$	$1\frac{3}{4}$	$1\frac{1}{2}$	$1\frac{1}{2}$
W	$6\frac{1}{2}$	$6\frac{1}{2}$	$6\frac{1}{8}$	$6\frac{1}{8}$	$5\frac{1}{2}$
X	$1\frac{1}{2}$	$1\frac{3}{4}$	2	$1\frac{3}{4}$	$1\frac{3}{4}$
Y	$5\frac{3}{4}$	$5\frac{3}{4}$	6	$5\frac{3}{4}$	$6\frac{1}{8}$
Z	$4\frac{3}{4}$	$4\frac{3}{4}$	$4\frac{3}{4}$	$4\frac{1}{2}$	$3\frac{7}{8}$

dimensions of the taper end. D_1 may then be calculated from the formula—

$$\frac{\pi}{4}(D_1^2 - D_2^2) - (D_1 - D_2)t = \frac{F}{f_t{}^1},$$

D_2 being first obtained by construction. Table II. gives the dimensions of locomotive crossheads of the type shown in fig. 52.

A round steel bar, whose diameter is 1 in., is loaded with 8 tons, and the end which is enlarged is cottered. Determine the thickness t of the cotter, its breadth b, and also the diameter D of the enlarged end of the rod, so that the stress in the cotter may be 7 tons, and the bearing pressure not more than 20 tons. Let d be diameter of the rod; then, if f is the stress per square inch,

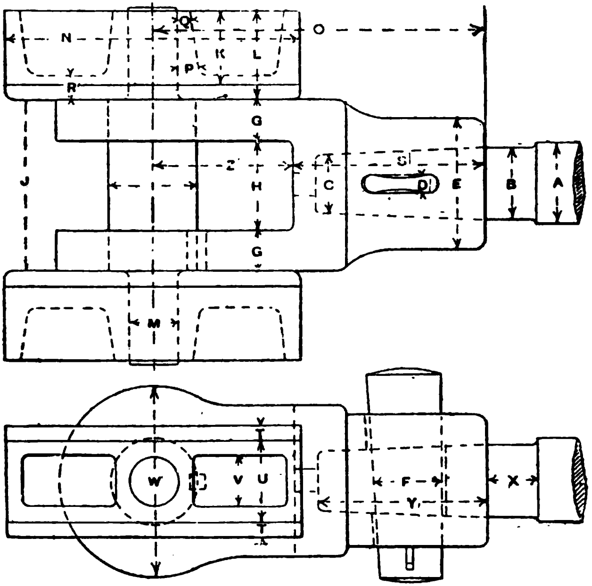


FIG. 52.

$$f d^2 \frac{\pi}{4} = f \left(D^2 \frac{\pi}{4} - D t\right) \quad . \quad . \quad . \quad . \quad (1)$$

$$= 8 \text{ tons},$$

$$f_s\, b\, t = 4 \text{ tons},$$

where f_s = shearing stress in the cotter.

$$7\, b\, t = 4 \text{ tons} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$20\, t\, D = 8 \text{ tons} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

From (1) and (3),

$$\frac{\pi}{4} = \frac{\pi}{4}\mathrm{D}^2 - \frac{8}{20},$$

$$\mathrm{D}^2 = \frac{1}{{\cdot}7854}\left({\cdot}7854 + \frac{8}{20}\right)$$

$$= 1{\cdot}51$$

$$\mathrm{D} = 1{\cdot}23.$$

From (3),

$$t = \frac{8}{20\,\mathrm{D}} = \frac{8}{20 \times 1{\cdot}23}$$

$$t = {\cdot}325$$

$$b = \frac{4}{7 \times {\cdot}325} = 1{\cdot}765.$$

To work out the proportions of a cottered joint, the rod having a tapered end, given that $f_t = 8{,}900$, $f_s = 9{,}500$, $f_t{}^1$ the stress in the boss $= 2{,}500$, $t = \frac{7}{18}\,b$, $\mathrm{P} = 3$ in. (See fig. 51 and Table I.)

$$2\,f_s\,b\,t = f_t\left(\frac{\pi}{4}\mathrm{D}_3{}^2 - \mathrm{D}_3\,t\right)$$

$$= f_t{}^1\left[\frac{\pi}{4}(\mathrm{D}_1{}^2 - \mathrm{D}_2{}^2) - (\mathrm{D}_1 - \mathrm{D}_2)\,t\right]$$

$$= f_t\,\frac{\pi}{4}\mathrm{P}^2.$$

$$\therefore\ 19000 \times \tfrac{7}{18}\,b^2 = 8900 \times {\cdot}7854 \times 9$$

whence

$$b = 2{\cdot}92\,;\ t = \tfrac{7}{18}\,;\ b = 1{\cdot}135\,;$$

$$f_t\,({\cdot}7854\,\mathrm{D}_3{}^2 - \mathrm{D}_3\,t) = 19000 \times \tfrac{7}{18} \times 8{\cdot}5\,;$$

$$\therefore\ {\cdot}7854\,\mathrm{D}_3{}^2 - 1{\cdot}135\,\mathrm{D}_3 = \frac{19000}{8900} \times \tfrac{7}{18} \times 8{\cdot}5,$$

whence $\mathrm{D}_3 = 3{\cdot}8$ very nearly.

Allowing for taper, let $\mathrm{D}_2 = 4$ in.

$$2500\,({\cdot}7854\,\{\mathrm{D}_1{}^2 - 16\} - \{\mathrm{D}_1 - 4\} \times 1{\cdot}135)$$

$$= 19000 \times \tfrac{7}{18} \times 8{\cdot}5,$$

whence $\mathrm{D}_1 = 7\ 42$

CHAPTER VI.

CONNECTING-ROD ENDS.

IN Tables III. and IV. will be found the dimensions of several locomotive connecting rods for cylinders of different sizes. We think this is preferable to giving proportional dimensions and average stresses. We have also given the diameter of cylinder stroke and pressure, and from these calculated the stresses at starting. These are not the working stresses, which depend upon forces with which at present we are not dealing, but they are probably the best aids in the design of the rod, as these forces are very difficult to calculate, and if mathematical formulæ for them were found, they would be quite unsuited to practical men. The following is the method of calculating these stresses in the case of the rod for the cylinder, 17 in. diameter, 24 in. stroke.

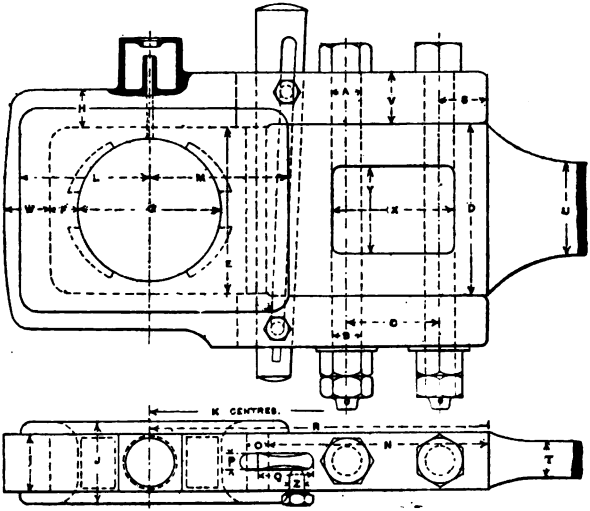


FIG. 53.

Figs. 53 and 54 show the large ends of all four rods, and the small ends of the last three. The load on the rod we have taken is—

$$150 \times 17^2 \times {\cdot}7854 = 34000 \text{ lb.},$$

neglecting obliquity of rod ; f_1 is the stress in the strap at the thinner part—

$$f_1 = \frac{34000}{2{\cdot}75 \times 1{\cdot}75 \times 2} = 3530 \text{ lb.}$$

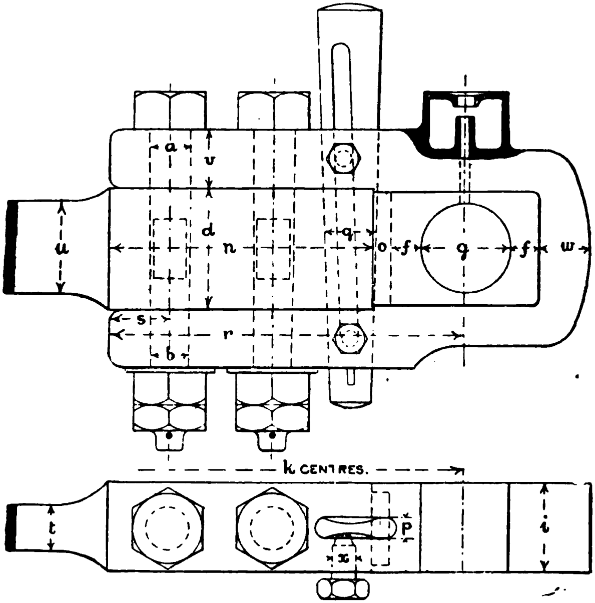


Fig. 54.

The mean diameter of the bolts is 1·312, and therefore the stresss f_2 in the strap at the bolts is—

$$f_2 = \frac{34000}{2{\cdot}5 \times 2 \times 1{\cdot}438} = 4725 \text{ lb.}$$

The shearing stress f_s in the bolts is—

$$f_s = \frac{34000}{4 \times \cdot 7854 \times 1 \cdot 312^2} = 6300.$$

There is also a bending stress in these bolts, but it is not of so much importance as the shearing stress. At the thinner end, where the bolt projects from the rod ends, its diameter is nearly 1¼ in.; and assuming that the load is uniformly distributed over the 2½ in. in the strap, the stress caused by bending is—

$$\text{F} = \frac{34000}{4} \times \frac{1\frac{1}{4} \times 32}{\pi \times (1\frac{1}{4})^3} = 5540.$$

(The modulus of a circular section is $\frac{\pi}{32} d^3$ where d is its diameter.)

This is less than f_s, and, as all cotters and pins in shear are more or less subject to bending, the mean shearing stress may be used for purposes of calculation rather than F. Moreover, it is proved in Cotterill's "Applied Mechanics," article 188, that the distribution of shearing stress is not uniform over a circular section, but at the diameter at right angles to the plane of bending is 1⅓ of the mean, so that the maximum shearing stress is 8,400 lb., and as this is greatest where there is no bending stress, and decreases as the bending increases, it is safe to take the larger stress of the two alone. In a rectangular section the greatest shearing stress is also at the centre, and is 1½ the mean. A hole is made in the rod end, which is circular or rectangular. This lightens the end. The section on either side of the hole is 2 in. by 2¾ in. broad, and as this is reduced by a bolt of mean diameter 1·312 in., the stress

$$f_5 = \frac{34000}{4 \times 1 \cdot 438} = 5900.$$

The strap is thicker where it crosses the centre line, as this portion has to resist a bending moment whose magnitude is unknown, because the distribution of pressure is unknown. If it were uniform, the case would be that of a beam, fig. 55, the load being upon 8⅛ in., the height of the brasses and the supports being taken at a distance apart equal to that of the centres of the upper and lower parts of the strap. Let F_1 be the stress, M the bending moment at the centre, and W the load.

$$M = \frac{W \times 9{\cdot}875}{4} - \frac{W \times 8{\cdot}125}{8}$$

$$= 1{\cdot}455\ W \text{ inch-pounds.}$$

The modulus of the rectangular section being $\frac{b \cdot h^2}{6}$ and

$$b = 2{\cdot}75$$

$$h = 2{\cdot}25$$

$$\therefore \quad F_1 = \frac{6\,M}{b\,h^2}$$

$$= \frac{1{\cdot}455 \times 34000 \times 6}{2{\cdot}75 \times (2{\cdot}25)^2}$$

$$= 21300 \text{ lb.}$$

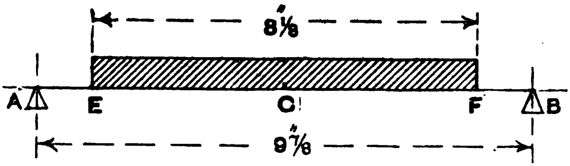


FIG. 55.

This is far greater than it could be reasonably expected to bear, and shows that the load must be distributed unequally, less in the centre, and more towards the ends E F.

$$f_3 = \frac{F_1}{6} = 3550.$$

It will be seen that in Table II.

$$\frac{W}{H} = 1{\cdot}335 \text{ to } 1{\cdot}19,$$

the mean value being 1·271.

If the shearing force at this section were $\frac{W}{2}$, the greatest possible, then the stresses would be 3,17, 2,870, and 3,380 in the four examples here given.

The stress in the large end of the rod itself is—

$$f_4 = \frac{33000}{T.U} = \frac{34000}{1\tfrac{11}{16} \times 4\tfrac{1}{2}} = 4470.$$

In the small end of the rod the stress is—

$$f_6 = \frac{34000}{3\frac{1}{8} \times 1\frac{5}{8}} = 6700.$$

TABLE III. (LARGE END).

Diameter of cylinder..	18	17	16	15
Stroke	26	24	24	20
Pressure	150	150	150	150
A	$1\frac{5}{8}$	$1\frac{3}{8}$	$1\frac{3}{8}$	$1\frac{1}{8}$
B	$1\frac{1}{4}$	$1\frac{1}{4}$	$1\frac{1}{4}$	1
C	3	$4\frac{1}{2}$	3	$2\frac{3}{4}$
D	$8\frac{5}{8}$	$8\frac{1}{4}$	$8\frac{1}{8}$	$6\frac{7}{8}$
E	$8\frac{1}{2}$	$8\frac{1}{8}$	8	$6\frac{3}{4}$
F	$1\frac{1}{4}$	$1\frac{3}{8}$	$1\frac{1}{4}$	$1\frac{1}{4}$
G	$7\frac{1}{2}$	7	7	6
H	$1\frac{1}{8}$	$1\frac{3}{4}$	$1\frac{3}{8}$	$1\frac{5}{16}$
I	3	$2\frac{3}{4}$	3	$2\frac{1}{2}$
J	4	$3\frac{15}{16}$	4	$3\frac{3}{4}$
K	77	$74\frac{1}{2}$	68	$54\frac{1}{2}$
L	$5\frac{7}{8}$	$6\frac{3}{8}$	$5\frac{5}{8}$	$4\frac{5}{8}$
M	$6\frac{5}{8}$	$6\frac{3}{4}$	$6\frac{3}{8}$	$4\frac{7}{8}$
N	$8\frac{5}{8}$	$10\frac{3}{4}$	$8\frac{5}{8}$	$7\frac{1}{2}$
O	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{3}{4}$	None
P	$\frac{7}{8}$	$\frac{7}{8}$	$\frac{7}{8}$	$\frac{3}{4}$
Q	$1\frac{7}{8}$	$2\frac{3}{16}$	$1\frac{7}{8}$	$1\frac{5}{8}$
R	$14\frac{3}{8}$	$16\frac{1}{2}$	$14\frac{1}{8}$	$11\frac{3}{4}$
S	$2\frac{1}{8}$	$2\frac{1}{2}$	$2\frac{1}{8}$	$1\frac{3}{4}$
T	$1\frac{3}{4}$	$1\frac{11}{16}$	$1\frac{1}{2}$	$1\frac{3}{8}$
U	$4\frac{1}{2}$	$4\frac{1}{2}$	4	$3\frac{5}{8}$
V	$2\frac{1}{2}$	$2\frac{1}{2}$	$2\frac{5}{8}$	$2\frac{1}{8}$
W	2	$2\frac{1}{4}$	$1\frac{3}{4}$	$1\frac{9}{16}$
X	$3\frac{5}{8}$	$5\frac{13}{16}$	$5\frac{3}{4}$	$4\frac{3}{4}$
Y	$3\frac{5}{8}$	$4\frac{1}{4}$	$3\frac{3}{8}$	$2\frac{5}{8}$
Z	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{5}{8}$
f_1	4235	3530	3650	4040
f_2	4500	4725	3760	4340
f_8	7050	6300	5575	7950
f_3	4560	3550	4410	5100
f_4	4840	4470	5030	5820
f_5	4500	5900	3870	4335

In the strap the stress is—

$$f_7 = \frac{34000}{3 \times 2 \times 1\frac{3}{8}} = 4120.$$

In the thicker part of the strap at the bolts

$$f_8 = 5120,$$

and f_9 is calculated for the small end in the same way as f_3 for the large.

In the fourth column of Table I. the engine referred to has a connecting rod whose large end is similar to that in fig. 53. Here the values of f_s, f_2, f_3, f_4 are 4,500, 4,260, 4,080, and 2,870. In the second column of the same table the connecting rod has a strap, gib, and cotter, and f_s, f_2, f_3, f_4 are 3,980, 2,780, 2,630, and 1,885.

TABLE IV. (SMALL END).

Diameter of cylinder	17	16	15
Stroke	24	24	20
Pressure	150	150	150
a	$1\frac{3}{8}$	$1\frac{5}{16}$	$1\frac{1}{16}$
b	$1\frac{5}{16}$	$1\frac{1}{4}$	1
c	$3\frac{1}{2}$	$2\frac{7}{8}$	$2\frac{1}{2}$
d	4	4	$3\frac{1}{4}$
e	$3\frac{7}{8}$	$3\frac{7}{8}$	$3\frac{1}{8}$
f	1	$\frac{3}{4}$	$\frac{5}{8}$
g	3	3	$2\frac{3}{8}$
h	$1\frac{3}{8}$	$1\frac{1}{8}$	1
i	3	3	$2\frac{1}{2}$
k	$74\frac{1}{2}$	68	$54\frac{1}{2}$
n	$8\frac{7}{8}$	$7\frac{1}{2}$	$6\frac{5}{8}$
o	$\frac{9}{16}$	$\frac{1}{2}$	None
p	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{5}{8}$
q	$1\frac{25}{32}$	$1\frac{1}{4}$	$1\frac{1}{8}$
r	$11\frac{15}{16}$	$10\frac{1}{4}$	$8\frac{9}{16}$
s	2	$1\frac{7}{8}$	$1\frac{5}{8}$
t	$1\frac{5}{8}$	$1\frac{1}{2}$	$1\frac{3}{8}$
u	$3\frac{1}{8}$	3	$2\frac{1}{2}$
v	2	$1\frac{5}{8}$	$1\frac{9}{16}$
w	$1\frac{3}{4}$	$1\frac{3}{8}$	$1\frac{1}{4}$
x	$\frac{3}{4}$	$\frac{5}{8}$	$\frac{5}{8}$
f_6	6700	6700	7710
f_7	4120	4460	5300
f_8	5120	5380	4900
f_9	3070	4060	4330
f_{10}	8140	8640	8925

In Table III. f_{10} is the stress at the centre of the rod when it is in compression. The method of calculating this will be explained later on. The value of f_{10} for the first rod in Table III. is not given in Table IV.; it is 8,150.

CHAPTER VII.

MARINE CONNECTING-ROD ENDS.

FIG. 56 shows a type of rod end used for both stationary and marine engines, but spoken of by the above name. In this bolts and a cap replace the strap. In Seaton's "Manual of Marine Engineering" the stresses are calculated by assuming the load on the rod to be the product of the boiler pressure and the area of the high-pressure cylinder. For wrought-iron bolts the stress is given as 5,000, and for steel 6,500, the weakest section of the bolt being at the bottom of the thread. In practice, however, the stress varies considerably, as may be seen by Table V., in which M. T. and M. C. are marine triples and compounds. U. S. N. refers to triples

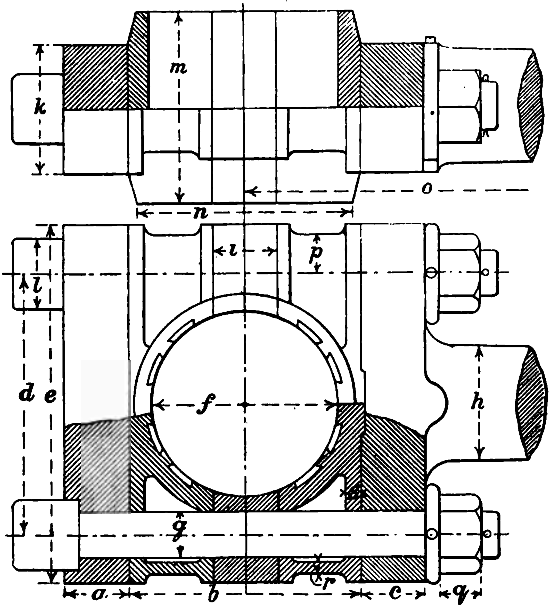


FIG. 56.

TABLE V.

Type of engine	M.T.	M.T.	M.T.	M.T.	M.T.	M.T.	M.T.	U S N	U S N	U S N	U S N	U S N	M.C.	M.C.	M.C.	M.C.	H.S.	H.S.	H.S.	H.S.
Boiler pressure	160	150	160	160	180	160	160	160	160	160	160	160	85	90	80	80	60	60	60	60c
Diameter of H.P. cylinder	8	11	19	$25\frac{1}{2}$	$31\frac{1}{2}$	27	$22\frac{1}{2}$	$34\frac{1}{2}$	32	42	25	$15\frac{3}{4}$	27	36	26	12	9	14	10	27
Stroke	16	18	36	42	54	48	36	42	42	42	36	24	36	42	36	16	18	24	20	36
Diameter of bolts	$1\frac{1}{4}$	$1\frac{5}{8}$	$2\frac{3}{8}$	$3\frac{7}{8}$	5	$4\frac{1}{8}$	$3\frac{1}{4}$	$4\frac{1}{4}$	4	5	3	2	$2\frac{7}{8}$	$3\frac{3}{4}$	$3\frac{1}{4}$	$1\frac{1}{2}$	1	$1\frac{1}{2}$	$1\frac{1}{8}$	$2\frac{1}{4}$
Length of rod	36	$40\frac{1}{2}$	$76\frac{1}{2}$	$94\frac{1}{2}$	120	105	72	84	84	84	72	48	72	$108\frac{1}{4}$	75	$32\frac{1}{2}$	48	60	50	90
Diameter at small end	$2\frac{1}{8}$	$2\frac{5}{8}$	$4\frac{1}{4}$	$6\frac{3}{4}$	8	7	$5\frac{7}{8}$	$6\frac{5}{8}$	$6\frac{1}{4}$	$8\frac{1}{8}$	5	$3\frac{1}{8}$	$4\frac{5}{8}$	$6\frac{1}{2}$	$4\frac{5}{8}$	$2\frac{1}{4}$	$1\frac{5}{8}$	$2\frac{1}{2}$	$2\frac{1}{4}$	$3\frac{3}{4}$
Diameter at large end	$2\frac{1}{2}$	$3\frac{1}{4}$	$4\frac{3}{4}$	$7\frac{3}{4}$	$9\frac{1}{2}$	$8\frac{1}{4}$	$7\frac{1}{2}$	$8\frac{1}{2}$	9	$10\frac{1}{2}$	$6\frac{3}{4}$	$4\frac{1}{4}$	$5\frac{3}{8}$	8	$5\frac{1}{2}$	$2\frac{1}{2}$	2	3	$2\frac{3}{4}$	*$4\frac{3}{4}$
f	4490	4850	6800	4560	4410	4760	5000	6600	6400	7000	7200	6770	4900	5260	3310	3480	3430	3550	3380	7000
f_1	2270	2620	3200	2290	2790	2380	2350	4325	4200	4000	4000	4060	2880	2760	2530	2270	1835	1885	1185	3730
f_2	2730	3000	4310	2580	3020	2680	2290	4060	3420	3730	3660	3790	3390	3080	2940	2710	3310	2865	1640	3800

* Diameter at middle $4\frac{3}{4}$ also.

in the United States Navy, and H. S. are horizontal stationary non-compound engines; one of these being a condensing engine, 12 lb. has been added to the boiler pressure. The stress in the bolt is f, the diameter at the bottom of the thread being that of a Whitworth bolt in all the columns except those marked U. S. N., in which the diameter is taken from a table of Sellers screws. The stress in the small end of the rod is f_1, and f_2 is the stress at the centre, calculated by a formula which takes into account the length of the rod, and which we shall explain presently.

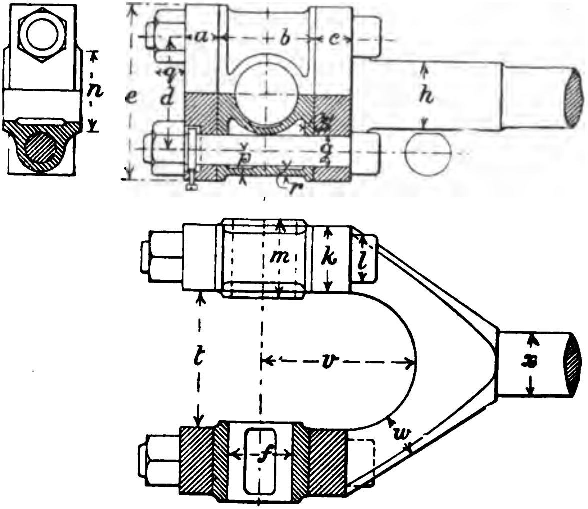


FIG. 57.

In the mercantile marine, connecting rods are generally forged of scrap iron, but in the navy forged steel is used, and the stresses allowed are generally much greater. Table VI. gives the dimensions of this type of rod end, the letters referring to those in fig. 56. The letters t, u, v, w refer to the white metal strips: t is the thickness, w is the number of strips, and $u\ v$ gives the widths outside and inside the brasses. These strips are cast in place, and project $\frac{1}{16}$ in. to $\frac{1}{4}$ in., and the "brasses" are frequently made of steel. In Table VI. they are made of brass.

The cap may be treated as a beam which has a span d, and is probably uniformly loaded. The stress f_3 (Table VI.) is calculated on this assumption. In the first four cases it does not differ much from 6,000 lb. The thickness c of the T end of the rod is generally made from ¼ in. to ½ in. thicker than the cap, as the load, when the rod is in tension, is concentrated near the centre. It is difficult to say what is the greatest stress in this part. If W is the load, f_4 has been calculated from the formula,

TABLE VI.

Diam. of H.P. Cyldr.	25½	31½	36	8	19	22½	27*	12
Stroke	45	54	48	16	36	36	36	16
Pressure	160	180	90	160	160	160	60c	80
a	4½	5½	4½	1¼	2¾	3½	2½	1⅝
b	16	21½	16	5¼	13¼	15¼	9	5¾
c	5	6	4½	1½	3	3⅝	2½	1½
d	17½	22½	18	5¾	13¼	18	9¼	6¾
e	24¾	32½	25	8½	17¼	24½	13¾	10
f	12½	16	13	4	9⅞	11	6½	4½
g	3⅞	5	3¾	1¼	2⅜	3¼	2¼	1½
h	7¾	9½	8	2½	4¾	7¼	4¾	2½
i	3¾	3½	3	⅝	3½	2	0	0
k	9	13	9	3½	6	8¾	5	4
l	5½	7½	5	1⅞	3⅜	5¼	3¼	2¼
m	12	16½	13¾	5	8	11⅛	8	4¾
n	16	21½	14½	5	12⅛	13¼	8½	5¾
o	94½	120	108	36	76½	72	90	32½
p	2¼	3¾	3	1	1½	2⅜	1⅝	1
q	2⅞	4	3	1	2¾	2½	2	1¼
r	⅜	¾	¾	solid	½	⅞	½	¼
s	1⅛	1½	1½	solid	1	⅞	1	⅝
t	⅝	¾	½	—	½	⅝	—	—
u	1⅝	2¼	2	—	3	1⅞	—	—
v	1⅞	2⅜	2⅛	—	3⅝	2	—	—
w	14	14	8	—	6	8	—	—
f_3	5860	6000	6775	6340	9900	8000	9500	6000
f_4	5350	5830	7530	5000	10700	9600	9200	6100
p	545	530	510	401	574	520	825	420

* Horizontal Stationary Condensing Engine.

$$f_4 \frac{k c^2}{6} = \frac{W(d - h)}{4}$$

The mean of the first four values is 5,927. We have not included the fifth and sixth columns, as the stresses in these rods are higher than is usual for the mercantile marine.

$$p = \frac{W}{f . m}$$

and is the pressure per square inch on the crank pin. It lies between 400 and 600 for marine engines.

TABLE VII.

Diameter of H.P. Cylinder	19	$25\frac{1}{2}$	$31\frac{1}{2}$	27
Stroke	36	45	54	48
Pressure	160	160	180	160
a	$2\frac{1}{8}$	$2\frac{1}{4}$	$3\frac{3}{4}$	$2\frac{3}{4}$
b	$5\frac{7}{8}$	$8\frac{1}{2}$	11	$8\frac{7}{8}$
c	$2\frac{1}{4}$	$3\frac{1}{4}$	4	$3\frac{1}{4}$
d	7	$11\frac{3}{8}$	$13\frac{1}{2}$	$11\frac{5}{8}$
e	11	$16\frac{5}{8}$	$21\frac{1}{4}$	$16\frac{7}{8}$
f	4	$6\frac{1}{2}$	8	6
g	2	$2\frac{3}{4}$	$3\frac{7}{8}$	$2\frac{7}{8}$
h	$4\frac{5}{8}$	$6\frac{3}{4}$	8	6
k	$4\frac{1}{8}$	$5\frac{1}{2}$	8	$5\frac{3}{4}$
l	$2\frac{5}{8}$	$4\frac{1}{4}$	$5\frac{1}{2}$	$4\frac{3}{8}$
m	$4\frac{7}{8}$	$6\frac{3}{4}$	$8\frac{3}{4}$	$6\frac{3}{4}$
n	5	$8\frac{1}{2}$	11	$8\frac{7}{8}$
o	$76\frac{1}{2}$	$94\frac{1}{2}$	120	105
p	$1\frac{1}{2}$	$1\frac{7}{8}$	$2\frac{7}{8}$	$1\frac{15}{16}$
q	2	2	$2\frac{7}{8}$	$2\frac{1}{8}$
r	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{2}$
s	solid	$\frac{7}{8}$	1	solid
t	$8\frac{1}{2}$	$13\frac{1}{4}$	$14\frac{1}{2}$	13
v	$9\frac{1}{4}$	$18\frac{1}{2}$	$20\frac{1}{4}$	20
w	$2\frac{7}{8}$	$3\frac{1}{2}$	$4\frac{3}{4}$	$3\frac{1}{2}$
x	$4\frac{1}{4}$	$6\frac{3}{4}$	8	7
f_5	4900	4430	3800	4600
f_6	6400	12600	6320	9000
f_7	4640	4910	4500	6100
p	1160	930	1000	1130
f_8	8320	5100	6100	7300

One form of small end for a marine connecting rod is shown in fig. 57, and Table VII. gives examples. The fork is generally longer, like that in fig. 58. The stress in the bolts is f_5, which should be less than that in the bolts at the large end, as the load may come more upon one pair than the other. The stress in the cap is f_6, calculated in the same way as f_3.

The stress f_7 in the T end is calculated from the formula,

$$f_7 \frac{k c^2}{6} = \frac{W}{2} \times \frac{d - h}{4},$$

which gives the stress due to bending at a distance $\frac{h}{2}$ from its centre—

$$p = \frac{W}{2 f m}$$

and is the pressure per square inch on the crosshead pins, which is from 900 to 1,200—

$$f_8 = \frac{p\, m^2}{\cdot 196 f^2}$$

being the stress caused by bending in these pins.

TABLE VIII.

Diam. of H. P. Cyldr.	8	26	22½	36	33½	27	12	10†
Stroke	16	36	36	48	39	27	16	20
Pressure	160	80	160	90	150	150	80	60
A	1½	3⅝	3⅞	4	3½	2¾	1¼	1⅜
B	3½	7	8¾	11	14	10¾	3¾	3⅝
C	2½	5½	6⅜	6¼	7½	6*	2¾	2⅜
D	6⅝	14½	18¾	19	19	12	6½	6¼
E	1⅛	3	3¼	3	2¾	2⅛	1⅛	1
F	2½	5¾	6	7	6¼	5⅛	2⅜	2⅝
G	4¾	10½	12	14	13½	10½	5	4⅜
H	2⅛	4¾	5⅞	6½	6	4⅞	2¼	2¼
p	920	1100	1140	1330	1260	1330	875	588
Revs.	170	88	70	72	120	140	—	—
f_1	1430	1230	1476	2170	3840	3930	1680	897
f_2	4270	4475	5170	9300	8570	8400*	3470	2745
f_3	1190	1255	1460	1480	3150	3460	1600	856

* Hollow gudgeon, internal diameter 3½ in. † Horizontal stationary engine.

Another form of small end for the connecting rod is shown in fig. 58. Dimensions are given in Table VIII., the fifth and sixth columns being taken from cruiser engines. In these it will be noticed that f_1, f_2, f_3 are very high, but p is not very different from the values in the other colums. If W is the whole pressure on the high-pressure piston, calculated as before, then

$$p = \frac{W}{B.C}$$

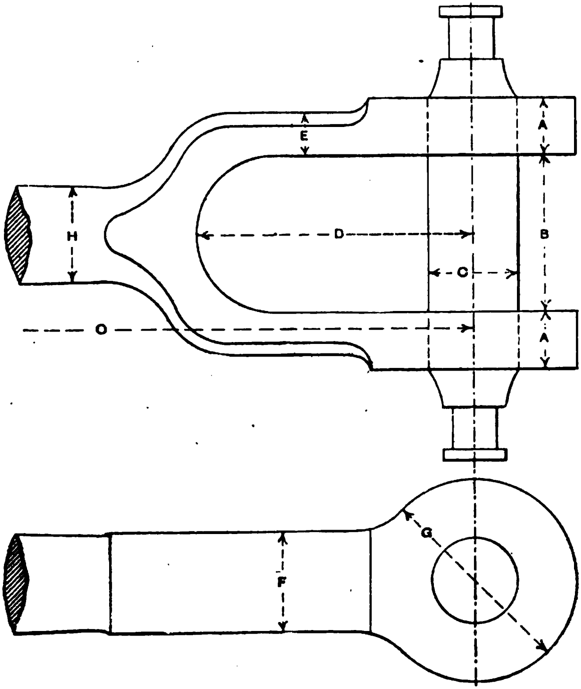


FIG. 58.

the bearing surface of the gudgeon being B.C, and p the pressure per square inch. The stress in the fork is

$$f_1 = \frac{W}{2\,E.F}$$

the section very being nearly E F at each side, the fact that it is turned making it differ slightly from this value.

$$f_2 \cdot \frac{\pi}{32} C^3 = \frac{W.(B + 2A)}{8}$$

gives the bending stress f_2 in the gudgeon, assuming that the load W is uniformly distributed over a length B, and that the reactions have their resultants at a distance B + A from one another—*i.e.*, are at the centres of the eyes.

The stress in the metal round the pin is

$$f_3 = \frac{W}{2A.(G - C)}.$$

If we try to work out proportional dimensions for this form of rod end, we shall find that we must assume different values of p, f, &c., for every type of engine. We have already shown that the engines of a cruiser, where high speed and lightness are required, weights are reduced as much as possible, and stresses are therefore much higher; but bearing surfaces cannot be reduced to any great extent. Suppose, then, that we have to design a rod end for an engine of the mercantile marine, we may proceed as follows:

Let $W = 2600 \times \cdot 7854 H^2$,

so that the stress in the small end of the rod when in tension is 2,600. Then

$$W = 2040 H^2.$$

Assume $F = 1 \cdot 2 H$, and $f_1 = 1500$;

then $E = \frac{2040 H}{3000 \times 1 \cdot 2} = \cdot 541 H.$

Assume $A = \cdot 67 H$,

$p = 1100$,

and $C = 1 \cdot 15 H.$

Then $BC = \frac{W}{p}$

$$\therefore B = \frac{2040 H}{1100 \times 1 \cdot 15} = 1 \cdot 61 H.$$

$$f_2 = \frac{4 W (B + 2A)}{\pi . C_3} = 5150.$$

If f_2 had been too great, it would have been necessary to increase C and decrease B. It is difficult to say when f_2 is too great, if we judge by examples from practice; but in all probability it should not exceed 5,500.

Gudgeons of case-hardened wrought iron have been found to work better than steel.

To find G, let $f_3 = 1500$.

$$\therefore 3000\ A\ (G - C) = 2040\ H^2$$

$$G - C = \frac{2040\ H}{3000 \times {\cdot}67}$$

$$= 1{\cdot}015\ H$$

$$G = 2{\cdot}165\ H = 1{\cdot}88\ C.$$

CHAPTER VIII.

On the Compression of Long Rods by Longitudinal Forces.

Figs. 59, 60, 61, and 62 show four ways in which compressive forces may act on a long bar. In fig. 59 the ends are fixed in the direction of the load, a practical example of this being a piston rod. In fig. 60 the ends are only guided in the direction of the load; in fig. 61 one end is fixed and the other guided in the direction of the load; in fig. 62 one end is free and the other fixed.

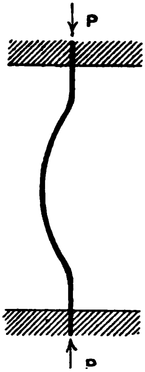


Fig. 59.

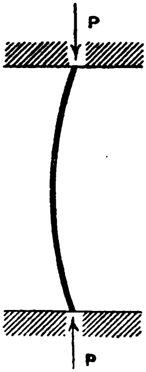


Fig. 60.

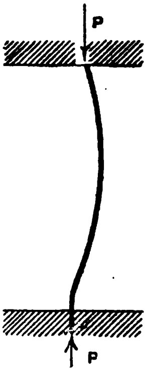


Fig. 61.

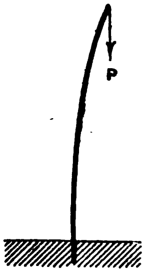


Fig. 62.

If E is the modulus of elasticity in pounds, l the length in inches, and P the breaking force in pounds, then in fig. 59, according to Euler, for rods of uniform section—

$$P = 4\,\pi^2 \frac{E\,I}{l^2};$$

in fig. 60, $$P = \pi^2 \frac{E\,I}{l^2};$$

in fig. 61, $$P = 2\,\pi^2 \frac{E\,I}{l^2};$$

in fig. 62, $$P = \frac{\pi^2}{4} \frac{E\,I}{l^2}.$$

I, in figs. 59 and 62, is the least moment of inertia of the section, and in figs. 60 and 61 is the moment of inertia about a line through the centre of area of the section, at right angles to the plane of bending.

The values of E are 29,000,000 for wrought iron, 30,000,000 for steel, and 17,000,000 for cast iron. For the safe load substitute n E for E, the corresponding values for this being 5,800,000, 6,000,000, and 2,800,000, according to Prof. Unwin.

This formula is, however, only suitable to very long rods, in which the increment of stress caused by bending is very much greater than the safe crushing stress per square inch. We can show this by taking an extreme case. Suppose, in fig. 60, that the depth of section is h and the breadth b, and that the plane of bending is in the direction of the depth. Then, if $b\,h$ is kept constant, and h increased, the value of I will increase, and P also; and as h may be increased to any extent, so also may P, which is absurd if $b\,h$ is constant. The rod may be supposed to be prevented from bending in the direction of the breadth by two smooth vertical planes.

The following formulæ (Gordon's, Seaton's, and Grashof's) are more suitable for practical purposes. Gordon's formula was modified by Rankine, and represents the results of an extensive series of experiments by Hodgkinson. Here W is the breaking load, l the length, A the area, f the coefficient of strength, h is the depth of section in the direction of bending, and n is a constant in the formula—

$$I = n\,A\,h^2;$$

all dimensions being in inches.

$$W = \frac{f\,A}{1 + \frac{l^2}{c\,n\,h^2}}.$$

The following table gives values of f and c:—

	f	c		
		Fig. 59.	Fig. 60.	Fig. 61.
Wrought iron	36,000	36,000	9,000	18,000
Steel (mild)	42,000	36,000	9,000	18,000
Cast iron	80,000	40,000	1,600	3,200

If the section is rectangular n is $\frac{1}{12}$, and if circular $\frac{1}{16}$.

In fig. 54, for a circular section

$$W = \frac{f A}{1 + a \frac{l^2}{d^2}}$$

where d is the diameter of section, and a is $\frac{1}{2250}$ for both wrought iron and steel. For a rectangular section

$$W = \frac{f A}{1 + a \frac{l^2}{h^2}}$$

where $a = \frac{1}{3000}$.

For fig. 60 change a to $4a$, and in fig. 61 a to $2a$, in the above equations.

Seaton's formula is only suitable for rods of circular section; P is the safe load usually taken as the boiler pressure multiplied by the area of the H.P. piston. S is the area of section, d is the diameter at the centre, and l is the length. Then for a piston rod which is in a similar condition to fig. 59,

$$P = \frac{f S}{1 + a \frac{l^2}{d^2}}$$

Seaton takes $f = 3000$,

and $a = \frac{1}{3000}$.

For connecting rods $R = \dfrac{f S}{1 + 4a \frac{l^2}{d^2}}$

as these are similar to fig. 60. R is the resultant of P and the pressure on the shoe, and for rods which are only twice the length of stroke

$$R = 1{\cdot}03\ P,$$

so that it is usual to take P instead of R.

Grashof's rules are as follow: P is the safe load, and f_1 f_2 are coefficients of strength, and A, I, l are the same as before. Then, for fig. 59,

$$P = \frac{f_1\,A}{1 + \dfrac{A\,l^2}{4\,C_1\,I}}$$

or

$$= \frac{f_2\,A}{1 + \dfrac{A\,l^2}{4\,C_2\,I}}$$

and for fig. 60

$$P = \frac{f_1\,A}{1 + \dfrac{A\,l^2}{C_1\,I}}$$

or

$$P = \frac{f_2\,A}{1 + \dfrac{A\,l^2}{C_2\,I}}$$

the lesser of the two values being always taken.

	C_1	C_2	f_1	f_2
Steel...............	5,000	5,000	12,000	12,000
Wrought iron...	5,600	5,600	10,000	10,000
Cast iron.........	10,000	2,400	3,000	12,000

For circular sections the above formulæ become—

$$P = \frac{f\,A}{1 + \dfrac{a\,l^2}{d^2}}.$$

For fig. 60

$$P = \frac{f\,A}{1 + \dfrac{4\,a\,l^2}{d^2}};$$

where a is $\frac{1}{1400}$ for wrought iron, and $\frac{1}{1250}$ for steel.

If the section is rectangular, and h the depth in the plane of bending, and b the breadth—

$$I = \frac{b\,h^3}{12};\ A = b\,h;$$

and in fig. 59
$$P = \frac{f\,A}{1 + \frac{3\,l^2}{C\,h^2}}.$$

In fig. 60
$$P = \frac{f\,A}{1 + \frac{12\,l^2}{C\,h^2}};$$

where C is 5,600 for wrought iron and 5,000 for steel.

CHAPTER IX.

CONNECTING-ROD ENDS.

IN Table V. f_2, and in Table IV. f_{10}, have been calculated from Gordon's formula, so that if P is the boiler pressure, D the diameter of the high-pressure cylinder, and d, l the mean diameter and length of the rod in Table V.,

$$f_2 = \frac{P\,D^2}{d^2}\left(1 + \frac{4}{2250}\,\frac{l^2}{d^2}\right)$$

and in Table IV.,

$$f_{10} = \frac{P\,\frac{\pi}{4}D2}{b\,h}\left(1 + \frac{1}{750}\,\frac{l^2}{h^2}\right)$$

where b h are breadth and depth of rod at centre.

In the columns in Table V. marked U.S N., the stress is slightly underestimated, as the rods, to save weight, are planed at the sides after turning, so that the width is reduced to that at the small end. This will not make much difference at the centre in the resistance to direct crushing, and far less in the resistance to bending, which depends on the moment of inertia of the section.

We can now show how to design a connecting rod for a marine engine. It will be best to take a numerical example. Let the pressure be 160 lb., diameter of high-pressure cylinder 20 in., length of rod 72 in., and diameter of crank pin 10 in. The length m of the bearing surface of the brasses may be calculated by assuming p from 520 to 550, where p is the pressure per square inch of bearing surface.

$$p\,m\,f = \text{W}$$

$$m = \frac{50200}{10\,p}$$

$$= 9{\cdot}66 \text{ to } 9{\cdot}15 \text{ in.}$$

Let $m = 9\tfrac{1}{4}$ in.

Then $p = 540$ lb.

Let the stress in the two bolts be 5,000 lb., and A be the sectional area at the bottom of the thread; then—

$$10000\ \text{A} = \text{W} = 50200.$$

$$\text{A} = 5{\cdot}02.$$

The value of A for a $2\tfrac{3}{4}$ in. bolt is 4·4637, and for a 3 in. bolt is 5·449. The mean of these, 4 956, will therefore be the section of a $2\tfrac{7}{8}$ in. bolt, which will make the stress a trifle over 5,000 lb. The pitch of the bolts is—

$$d = f + g + 2\,x$$

where x = thickness of brasses or packing piece between pin and bolt,

= $\tfrac{9}{16}$ in. for a $12\tfrac{1}{2}$ in. pin.
$\tfrac{1}{2}$ in. for a $9\tfrac{7}{8}$ in. pin.
$\tfrac{1}{4}$ in. for a $6\tfrac{1}{2}$ in. and 4 in. pin.

Here, then, $2\,x$ will be 1 in., and

$$d = 13\tfrac{7}{8}.$$

The diameter H of the small end of the rod is given by

$$\text{H}^2 = \frac{\text{P}\,\text{D}^2}{f_1}$$

where P = boiler pressure = 160 lb.
D = cylinder diameter = 20 in.

and f_1 may be taken between 2,300 and 2,800; if $f_1 = 2{,}600$,

$$\text{H} = \sqrt{\frac{160 \times 20^2}{2600}}$$

$$= 5 \text{ in. very nearly.}$$

The diameter H_1 at the centre may be calculated from Gordon's formula,

$$f_2 = \frac{\text{P}\,\text{D}^2}{\text{H}_1{}^2}\left(1 + \frac{4\ \text{L}^2}{2250\ \text{H}_1{}^2}\right)$$

where L is the length of rod.

This may be thrown into the form—

$$f_2 \, . \, H_1^4 - P D^2 H_1^2 - \frac{4}{2250} L^2 \, . \, P D^2 = O$$

$$H_1^2 = P D^2 \, . \, \frac{1 + \sqrt{1 + \frac{16}{2250} f_2 \frac{L^2}{P D^2}}}{2 f_2}$$

$$= \frac{P D^2}{6000} \left(1 + \sqrt{1 + 21{\cdot}\dot{3} \, \frac{L^2}{P D^2}} \right)$$

if $f_2 = 3000.$

It is easy to manage the above formula *with a slide rule*, although there are two square roots to be worked out.

$$H_1 = 5{\cdot}32.$$

Then, since $H = 5,$

$$h = 5{\cdot}64 = 5\tfrac{5}{8} \text{ in.},$$

because the rod is conical.

There is no fixed rule for k. In Table VI. the values of k/h are 1·16, 1·37, 1·125, 1·4, 1·26, 1·2, 1·05, and 1·6. Suppose k/h is 1·15 in this case, then k will be 6½ in.

To calculate c, assume $f_4 = 7500$; then

$$c = \sqrt{\frac{W \, (d - h)}{5000 \, k}}$$

$$= 3{\cdot}57 = 3\tfrac{5}{8}, \text{ say.}$$

To calculate a, assume

$$f_3 = 7500$$

$$a = \sqrt{\frac{W \, d}{1000 \, k}}$$

$$= 3{\cdot}29 \text{ in.} = 3\tfrac{1}{4} \text{ in. very nearly.}$$

The dimension b can be taken from Table VI. The nearest diameter of crank pin to the one we are considering is 9⅞ in., and for this $b = 13\frac{1}{4}$ in., so that in this case we may make

$$b = 13\tfrac{3}{8}.$$

The white metal strips may be taken from the same column. They will be six in number, each ½ in. thick, 3 in. broad outside, and 3⅝ in. inside the brasses. They may stand out $\frac{1}{4}$ in. from the brasses, and may be held more firmly in the brasses by a dovetail at the back, $\frac{3}{8}$ in. thick, increasing in breadth from $1\frac{1}{4}$ in. to $1\frac{1}{2}$ in.

In stationary engines we frequently find connecting rods enlarged in the middle. A few examples are given in

TABLE IX.

Type..	H.S.	H.S.	H.S.	H.S.	H.S.
H.P. cylinder	$14\frac{1}{2}$	13*	40	24†	32
L.P. cylinder	—	$22\frac{1}{2}$	—	46	60
Stroke	30	24	120	72	84
Boiler pressure	80*c*	120*c*	80*c*	90*c*	100*c*
Length of rod	90	60	316	180	210
Diameter at small end	$2\frac{3}{8}$	$2\frac{1}{2}$	8	6	7
Diameter at middle	4	$3\frac{1}{8}$	12	8	$9\frac{1}{2}$
Diameter at large end	$3\frac{1}{4}$	$2\frac{3}{4}$	9	6	$7\frac{1}{2}$
f_1	3430	3400	2300	2100	2080
f_2	2140	3485	2285	2400	2120
f_3	1840	2810	1820	2100	1810
Max. pressure H.P.C.	—	65	—	73	77
Max. pressure L.P.C.	—	42	—	16	23

* Cranks at right angles. † Tandem.
H.S. = horizontal stationary.

Table IX. The greatest possible load has been taken in each case for calculating the stresses f_1 at the small end, f_3 at the large end, and f_2 at the centre, which last has been obtained by Gordon's formula.

CHAPTER X.

PISTON RODS.

GORDON'S formula may be used for calculating the diameter of a piston rod. In some cases it may be treated as if it is fixed at both ends in the direction of the load, a marine-engine rod being an example of this, and in others as if it is fixed at one end, and at the other merely guided in the direction of the load. Figs. 59 and 61 illustrate these two cases. The piston end is the fixed end, while the crosshead end may only furnish a pin joint, as in the case of the locomotive crosshead, fig. 52.

In the former case, if f_2 is the stress in the rod,

$$f_2 = \frac{p\,D^2}{d^2}\left(1 + \frac{1}{2250}\,\frac{l^2}{d^2}\right)$$

and in the latter

$$f_2 = \frac{p\,D^2}{d^2}\left(1 + \frac{1}{1125}\,\frac{l^2}{d^2}\right)$$

In engines for warships f_2 may be as high as 4,500 lb., but in the mercantile marine we have found it as low as 2,200 lb., and rarely more than 3,000 lb. The stress f_1 in the screwed end (Table X.) may lie between 7,000 and 8,500 lb. for warships, while in the mercantile marine it does not much exceeed 5,000 lb., and is usually less.

In Table X., U.S.N. refers to cruisers in the United States Navy, all triple-expansion engines; H.M.S. to the same in the British Navy; M.T., M.C. are marine triples and compounds; H.S. stands for horizontal stationary engines; L. for locomotives, and V.P. for vertical pumping engines; H.T.S. for horizontal triple-expansion stationary engine, and P. for portable engine. The letter *c* against the boiler pressure means that the engine is a simple condensing engine, and 12 lb. is added to the pressure in calculating the stresses. Where the latter equation for f_2 must be used, the mark † has been placed against the diameter of the piston rod. In the line "Length and breadth of shoe," if there are two side blocks, twice the width of one has been given. No one has yet given any satisfactory rule for calculating the sizes of these, and experience is our only guide. If the rod is fixed at both ends in the direction of the load, its length is measured from the point where it leaves the piston to the

TABLE X.

Type	U.S.N.	U.S.N.	U.S.N.	U S.N.	U.S.N.	H.M.S.	H.M.S.	H.M.S.	M.T.	M.T.	M.T.	M.C.	M.C.	L.
Diameter of H P. cylinder	$34\frac{1}{2}$	32	42	25	13	27	30	43	$19\frac{1}{2}$	8	$5\frac{1}{2}$	36	26	$17\frac{1}{2}$
Diameter of L.P. cylinder	..	..	..	..	..	..	..	..	..	..	..	..	..	..
Stroke	42	42	42	36	24	27	36	51	36	16	8	48	36	26
Boiler pressure	160	160	160	160	160	150	135	135	150	160	160	90	80	150
Length of rod	66	63	63	53	34	44	55	74	55	$27\frac{1}{2}$	6	68	44	87
Diameter of rod	$6\frac{5}{8}$	$6\frac{1}{4}$	$8\frac{1}{8}$	$4\frac{7}{8}$	$2\frac{5}{8}$	5	6	$8\frac{1}{2}$	$5\frac{1}{4}$	$2\frac{1}{8}$	$1\frac{1}{4}$	$6\frac{1}{2}$	$4\frac{3}{4}$	$2\frac{3}{4}$
Diameter of screwed end	$5\frac{1}{2}$	$5\frac{1}{4}$	7	4	$2\frac{1}{4}$	4	$4\frac{1}{2}$	$6\frac{1}{2}$	..	$1\frac{3}{4}$	..	$5\frac{1}{4}$	$4\frac{1}{8}$	$2\frac{1}{2}$
Length and breadth of shoe	$23\frac{1}{2} \times 19$	22×17	$26\frac{1}{2} \times 24$	$17\frac{1}{2} \times 13$	10×7	..	19×17	28×23	..	$7 \times 3\frac{3}{4}$	..	$18\frac{1}{4} \times 15$	18×13	14×5
f_1	7760	7300	7000	7850	7000	8500	7400	7000	..	4600	..	5200	4000	9700
f_2	4520	4380	4370	4410	4220	4520	3500	3600	2170	2440	3300	2880	2190	6550
Length of connecting rod	..	..	..	..	..	..	..	..	..	..	..	..	..	..

U.S.N. = United States Navy. H.M.S. = Her Majesty's Navy. M.T. = Marine Triples. M.C. = Marine Compounds.
L = Locomotive.

TABLE X.—CONTINUED.

Type	L.	P.	H.T.S.	H.S.	H.S.	H.S.	H.S.	H.S.	H.S.	H.S.	H.S.	H.S.	H.S.	V.P.
Diameter of H.P. cylinder	18	7	16	40	22¼	27	28	9	10	14½	14	13	6½	28
Diameter of L.P. cylinder	..	..	..	..	..	..	..	..	..	..	..	22½	..	52
Stroke	26	10	66	120	40	36	42	18	20	30	24	24	8	78
Boiler pressure	140	100	150	80c	40c	60c	60c	60c	60	80c	60	42*	100	60
Length of rod	45	24	94	144	..	66	76	25	31	40	34	45	..	127
Diameter of rod	2¾†	1⅝	4½†	8	3	3½†	4†	1½	2⅛	2⅜	2	2½†	1¼	5†
Diameter of screwed end	2⅝	1¼	..	6	..	4	3½	1¼	1⅞	2	..	2½	1	4
Length and breadth of shoe	10×6½	6½×3½	..	39×15	..	14×10	14×10	10×6	10×6	10½×6	12×4½	10×7½	7×3¾	..
f_1	8750	4300	..	4900	..	4100	5900	5100	2370	6600	..	4500	6000	3690
f_2	7425	2040	3050	2630	2860	5620	4650	2910	1450	3850	3330	4370	2700	2950
Length of connecting rod	..	48	..	316	96	90	99	48	50	90	60	60	24	..

* Pressure from diagram on L.P. piston. † These to be treated as held by pin joint at one end.

L = Locomotive. H.T.S. = Horizontal triple-expansion stationary. P = Portable. H.S. = Horizontal stationary.
V.P. = Vertical pumping.

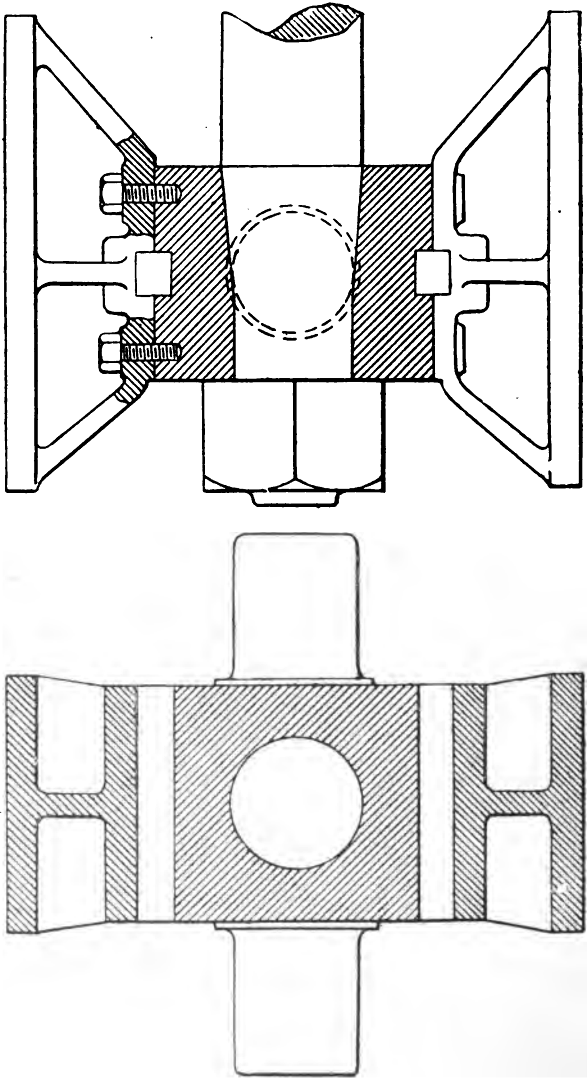


Fig. 63.

point where it enters or is forged to the crosshead; if there is a pin joint, as in fig. 61, its length is from the piston to the centre of this joint. In the marine engines given in the table, the connecting rod is about twice the stroke, and in the locomotives about three times; the lengths have been given for the remainder.

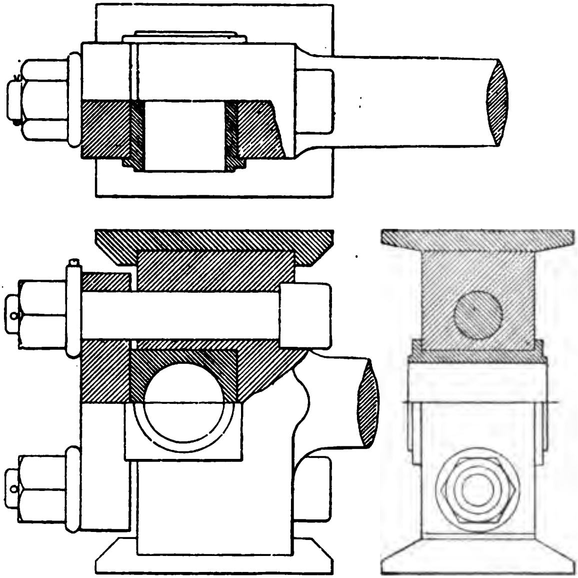


FIG. 64.

In figs. 63 and 64 two forms of marine crossheads are shown suitable to the connecting rod ends in figs. 57 and 58.

CHAPTER XI.

STRESSES CAUSED BY BENDING COMBINED WITH TENSION OR COMPRESSION.

SUPPOSE a force P acts parallel to a rod B C, fig. 65, then two equal and opposite forces P may be placed at C, as shown, without causing any alteration. We therefore have a couple P r and a force P acting on the section at B, whose moduli are Z_t Z_c for tension and compression and area A. Then the stress caused by the direct pull on the section is

$$f_1 = \frac{P}{A},$$

and the two stresses caused by bending are f_t f_c, where

$$f_t = \frac{Pr}{Z_t};\ f_c = \frac{Pr}{Z_c}.$$

The total tensile stress is therefore $f_1 + f_t$ at the right hand, and the total compressive stress is $f_c - f_1$. In fig. 66 the action that takes place is shown graphically. It depends on the form of the section whether the tensile or compressive stress is the greater.

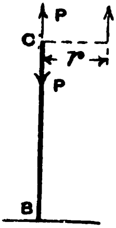


FIG. 65.

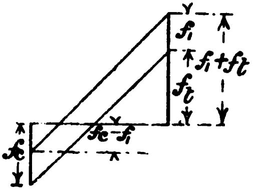


FIG. 66.

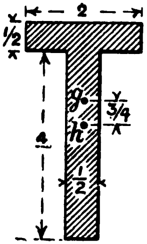


FIG. 67.

If the forces in fig. 65 are reversed, then

$$f_c = \frac{Pr}{Z_t} \text{ and } f_t = \frac{Pr}{Z_c},$$

but f_1 is greater than $\frac{P}{A}$, because the rod is in compression, and must be calculated by Gordon's formula.

If at any section of any part of a machine there is a bending moment M, and a longitudinal pull P, acting through the centre of area of the section, then the stresses are

$$f_2 = \frac{\mathrm{P}}{\mathrm{A}} + \frac{\mathrm{M}}{Z_t} \text{ in tension,}$$

and

$$f_3 = \frac{\mathrm{M}}{Z_c} - \frac{\mathrm{P}}{\mathrm{A}} \text{ in compression.}$$

If P is compressive, and f_1 is the stress calculated by Gordon's formula, then

$$f_2 = \frac{\mathrm{M}}{Z_t} - f_1 \text{ in tension,}$$

$$f_3 = \frac{\mathrm{M}}{Z_c} + f_1 \text{ in compression.}$$

The following are numerical examples of the above equations :—

Example I.—A T piece of iron, fig. 67, is subjected to a longitudinal pull, which passes through the centre of depth of the web. Determine the ratio of the greatest stress to that which would be obtained if the line of action of the pull passed through the centre of area of the T section.

It may be easily shown that the centre of area g is $2\frac{3}{4}$ in. from the end of the web, so that the moment of the pull P is P . $g\,h$ where $g\,h = \frac{3}{4}$ in.

The greatest tensile stress will evidently be greater than the compressive, and will be at the end of the web. Also Z_t = 2·2 inch-units ;

$$\therefore f = f_1 + f_t = \mathrm{P}\left(\frac{1}{\mathrm{A}} + \frac{g\,h}{Z_t}\right)$$

$$= \mathrm{P}\left(\tfrac{1}{3} + \frac{\cdot 75}{2\,2}\right) = \cdot 674\,\mathrm{P}.$$

If P had acted through the centre of area of the section, the stress would have been

$$\frac{\mathrm{P}}{\mathrm{A}} = \frac{\mathrm{P}}{3}.$$

The ratio required is therefore 2 022.

Example II.—A steel stay for a marine boiler is 19 ft. 2 in. long and 3 in. diameter. The pressure is 160 lb. per square inch, and the area supported by the stay is 306 square inches at each end. It is held to the end plate by nuts, with washers inside and outside. To find the stress in the stay, its weight per cubic inch being ·29 lb., and the sectional area at the bottom of the thread being 5·45 square inches, and its diameter 2 635 in.

We must here consider two possible cases. Firstly, we may treat the stay as a beam *fixed horizontally* at both ends, and, secondly, as a beam merely supported at both ends. The greater stress should then be allowed for. In the first case the bending moment at the screwed end is $\frac{Wl}{12}$, where W is the weight and l the length of stay.

$$M = \frac{Wl}{12} = \frac{{\cdot}7854 \times 3^2 \times 230 \times {\cdot}29 \times 230}{12}$$

$$= \frac{471{\cdot}5 \times 230}{12} = 9036{\cdot}6 \text{ inch-pounds.}$$

$$Z = \frac{\pi}{32} \times (2{\cdot}635)^3$$

$$f_t = \frac{M}{Z} = \frac{9036{\cdot}6 \times 32}{\pi \times (2{\cdot}635)^3} = 5050, \text{ in round numbers.}$$

$$f_1 = \frac{P}{A} = \frac{160 \times 306}{5{\cdot}45} = 8983.$$

$$\therefore f_2 = \frac{P}{A} + \frac{M}{Z}$$

$$= 5050 + 8983 = 14033 \text{ lb.}$$

In the second case the greatest moment would be at the centre.

$$M = \frac{Wl}{8} = 13555$$

$$Z = \frac{\pi \times 3^3}{32} = 2{\cdot}6\dot{7}$$

$$f_t = \frac{13555}{2{\cdot}6\dot{7}} = 5062$$

$$f_1 = \frac{P}{A^1} = \frac{160 \times 306}{{\cdot}7854 \times 9}$$

$$= 6926$$

$$\therefore f_2 = 6926 + 5062 = 11988 \text{ lb.}$$

So that 14,033 lb. is the greatest possible stress.

The Board of Trade allow 9,000 lb. per square inch stress, and neglect the effect of bending, which appears, however, from the above case to increase it considerably.

Example III.—A locomotive coupling rod is 7 ft. 9 in. between centres; it is of uniform section, $3\frac{3}{4}$ in. deep and $1\frac{3}{8}$ in. broad. The crank is 12 in., and the revolutions 200 per minute. It is of wrought iron, and weighs ·28 lb. per cubic inch. Find the total stress when it is in its lowest position and in compression, the force it transmits being 16,500 lb.

We have assumed that it transmits half the pressure of one piston, which is calculated thus: Boiler pressure 150, back pressure 4 lb, diameter of cylinder 17 in., give whole pressure on piston 33,000 lb., in round numbers. This is probably somewhat more than it would have to bear at 200 revolutions, because the cut-off would be before half stroke. The greatest bending stress comes on the rod when at its lowest position, because the motion of each point relative to the engine is a circle, and the centrifugal force is acting downwards (fig. 68), at right angles to A B, and the bending is also slightly increased by the weight of the rod itself. Let V be the velocity of each point on the rod, W its weight, and r the crank radius in feet; then the bending moment at the centre caused by this uniform load is

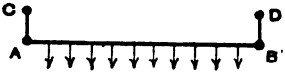


FIG. 68.

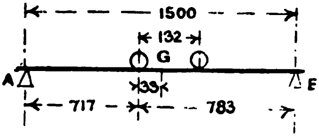


FIG. 69.

$$M = \frac{\left(W + \frac{W V^2}{g r}\right) l}{8} \text{ inch-pounds,}$$

where l = A B in inches.

Let b, h be breadth and depth of the rod; then

$$f_c = \frac{\left(W + \frac{W V^2}{g r}\right) l}{8 \times \frac{b h^2}{6}}$$

$$= \frac{\left(\cdot 28\, b\, h\, l + \frac{\cdot 28\, b\, h\, l \left(2\, \pi\, r\, \frac{\mathrm{N}}{60}\right)^2}{g\, r}\right) l}{\frac{4}{3}\, b\, h^2}$$

$$= \cdot 21\, \frac{l^2}{h}\left(1 + \cdot 00034\, r\, \mathrm{N}^2\right) = 7060 \text{ inch-pounds.}$$

It will be noticed that the breadth of the rod does not affect the bending stress, because, although it increases the strength, it also increases the load to the same extent. To find f_1, we use Gordon's formula—

$$f_1 = \frac{\mathrm{P}}{b\, h}\left(1 + \frac{l^2}{c\, n\, h^2}\right)$$

where $c = 9000,\ n = \frac{1}{12}$;

$$\therefore f_1 = \frac{16500}{1 \cdot 375 \times 3 \cdot 75}\left(1 + \frac{93^2 \times 12}{9000 \times 3 \cdot 75^2}\right)$$

$$= 3200 \times 1 \cdot 82 = 5820\,;$$

$$\therefore f_1 + f^c = 5820 + 7060 = 12880.$$

Example IV.—Fig. 69A shows a wall crane carrying $1\frac{1}{2}$ ton; AB, the jib, has a section whose area is $11\frac{1}{2}$ square inches, and moment of inertia 33 in. units; while the distances from the top and bottom of the section to the centre of area are $2\frac{3}{4}$ in. and $3\frac{3}{4}$ in. respectively. The monkey, or traveller, that carries the load has its two axles 12 in. apart.

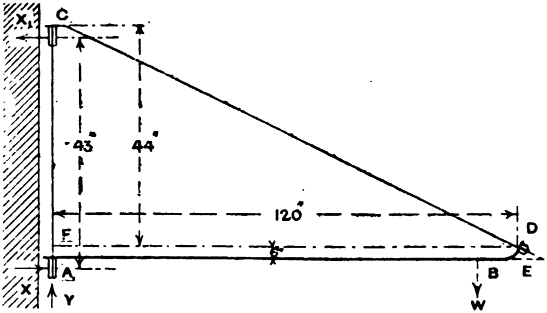


FIG. 69A.

Find the position of load that will cause the maximum bending moment on the jib A B, and find the tensile and compressive stresses when it is in this position.

$$\text{A E} = 120 \times \frac{50}{40} = \frac{1500}{11}.$$

We have already shown that maximum bending is produced when one pair of wheels is placed at a distance from the centre of the beam equal to one quarter of the distance apart of the axles.

The stresses in the straight part of A B will not be altered if we replace the curve B D by the two straight lines B E, D E, so that the beam we have to deal with is shown in fig. 69, the dimensions being given in elevenths of an inch. The left wheel is placed 3 in. from the centre, as this brings the load nearer E, and the force at E must therefore be greater than it would be if the right wheel were 3 in. from the centre. This will increase the compressive stress, but decrease the tensile stress, which latter would be greater if the right wheel were nearer the centre.

The reaction at E is

$$\text{F} = 1\tfrac{1}{2} \times \frac{783}{1500} = \cdot783,$$

and the horizontal component of the stress in C D is

$$\text{P} = \cdot783 \times \frac{120}{44} = 2{\cdot}14 \text{ tons.}$$

$$f_1 = \frac{\text{P}}{\text{A}} = \frac{2{\cdot}14}{11\tfrac{1}{2}} = \cdot186 \text{ ton.}$$

The vertical force F_1 at A is ·717 ton, so that the bending moment at the left wheel is

$$\text{M} = \cdot717 \times \frac{717}{11} = 46{\cdot}75 \text{ inch-tons}$$

$$Z_t = \frac{\text{I}}{3\tfrac{3}{4}} = 8{\cdot}8$$

$$Z_c = \frac{\text{I}}{2\tfrac{3}{4}} = 12$$

The bending stresses are

$$f_t = \frac{\text{M}}{Z_t} = \frac{46{\cdot}75}{8{\cdot}8} = 5{\cdot}32$$

$$f_c = \frac{\text{M}}{Z_c} = \frac{46{\cdot}75}{12} = 3{\cdot}9, \text{ nearly.}$$

The total tensile stress is therefore

$$f_2 = f_t - f_1 = 5{\cdot}32 - {\cdot}186 = 5{\cdot}134 \text{ tons},$$

and the compressive stress is

$$f_3 = f_c + f_1 = 3{\cdot}9 + {\cdot}186 = 4{\cdot}086 \text{ tons}.$$

☞ Strictly speaking, we should have applied Grashof's or Gordon's formula to find f_1, as the stress is compressive.

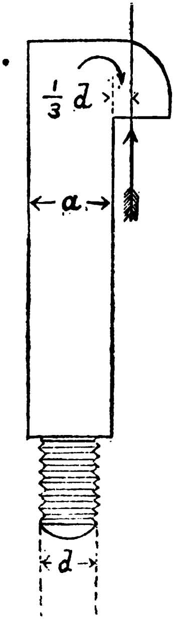


Fig. 70.

The jib must be treated as if it has pin points at both ends, and the moment of inertia must be taken about a horizontal axis.

Supposing the section of the shank of the bolt, fig. 70, is a rectangle $a \times d$, and the line of action of the force exerted on the hooked head to be as shown in the figure, determine the ratio of the greatest stress in the shank to the mean stress on the screwed portion of the bolt at the bottom on the thread in terms of a and d, and determine the ratio of a to d for uniformity of strength.

Let P be the force,

$$P = f\,\frac{\pi}{4} d^2 .$$

$$P\left(\tfrac{1}{3} d + \frac{a}{2}\right) = f\,\frac{d\,a^2}{6} .$$

$$\therefore \frac{d}{3} + \frac{a}{2} = \frac{4\,d\,a^2}{6\,\pi\,d^2} = \frac{2}{3\,\pi} \cdot \frac{a^2}{d} .$$

$$\therefore d^2 + \tfrac{3}{2}\,a\,d - {\cdot}6375\,a^2 = 0.$$

$$d = {\cdot}345\,a.$$

Within certain limits, the looseness of the fit of a piston in its cylinder, and the sides of the guide block in the guides, will permit the ends of the piston rod to undergo free angular deviation from the originally straight axis, so that, within these limits, a piston rod under compression should be considered as in the condition of a pillar freely jointed at its ends. Suppose the ordinates of the deviation curve to be proportional to a curve of sines of angles from 0 to π, show that at the limit, when the ends of the rod are restrained from further angular deviation.

$$\frac{p^1}{p_0} = \frac{8}{\pi} \cdot \frac{L}{l} \cdot \frac{v}{d} .$$

Where p_0 is the intensity of the uniform compressive stress due to the pressure on the piston, p^1 is the increase of stress due to lateral bending, L is the length of the piston rod, l is the length of the guide-block surface or depth of piston where it fits the cylinder, and v is the amount of the looseness of fit of the piston in the cylinder, or side of guide block in the guides.

In a curve of sines, let

$$y = m \sin \frac{x\,\pi}{L} ;$$

then
$$y = m \text{ when } x = \frac{L}{2} ;$$

$$\frac{d\,y}{d\,x} = \frac{m\,\pi}{L} \cos \frac{x\,\pi}{L} .$$

Hence the tangent of inclination of the curve to the axis of x when $x = o$ is

$$\left(\frac{d\,y}{d\,x}\right)_0 = \frac{m\,\pi}{\mathrm{L}}.$$

Now, it is evident that this is $\frac{v}{l}$;

$$\therefore\ \frac{v}{l} = \frac{m\,\pi}{\mathrm{L}}$$

$$\therefore\ m = \frac{v\,.\,\mathrm{L}}{l\,.\,\pi}.$$

Now, if P is the load on the piston,

$$\mathrm{P} = p_0\,.\,\frac{\pi}{4}\,d^2,$$

and

$$\mathrm{M} = \mathrm{P}\,m\,. = p^1\,\frac{\pi}{32}\,d^3;$$

$$\therefore\ p^1 = \frac{p_0\,\frac{\pi}{4}\,d^2\,m}{\frac{\pi}{32}\,d^3}$$

$$= \frac{8\,p_0\,m}{d}.$$

$$\therefore\ \frac{p^1}{p_0} = \frac{8}{\pi}\,.\,\frac{\mathrm{L}}{l}\,.\,\frac{v}{d}.$$

CHAPTER XII.

SHAFTING.

IF a couple T, fig. 71, act at right angles to the axis of a shaft, it produces torsion. If fig. 72 be any section, and a ring of small thickness t and mean radius r be taken, there is a shearing stress per square inch in this ring equal to $k\,r$ where k is some constant, and therefore the total moment of the stresses in this ring about the axis of the shaft is

$$k\,r\,.\,2\,\pi\,r\,t\,.\,r\,. = 2\,\pi\,k\,r^3\,t.$$

The moment of all the stresses on the section is therefore $\Sigma\,2\,\pi\,k\,r^3\,t$, the symbol Σ meaning the summation of the moments of all the rings, and this must equal T.

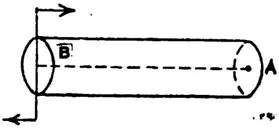


FIG. 71.

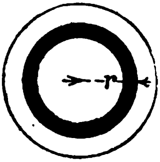


FIG. 72.

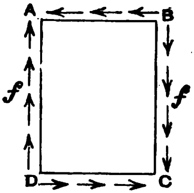


FIG. 73.

$$\therefore\ \mathrm{T} = 2\,\pi\,k\,\Sigma\,r^3\,t$$

$$= \frac{\pi\,k\,r_1{}^4}{2} = \frac{\pi\,f\,r_1{}^3}{2},$$

because $k\,r_1 = f$, the shearing stress at the outer radius r_1;

$$\therefore\ \mathrm{T} = \frac{f\,d^3}{16} = \cdot196\,f\,d^3$$

if d = the diameter of shaft.

If the shaft be hollow and the internal radius be r_2, then

$$\mathrm{T} = \frac{\pi\,k\,(r_1{}^4 - r_2{}^4)}{2} = \frac{\pi\,f}{2\,r_1}\,(r_1{}^4 - r_2{}^4)$$

$$= \frac{\pi\,f}{16}\cdot\frac{d_1{}^4 - d_2{}^4}{d_1},$$

where d_1 d_2 are the external and internal diameters.

The above values of f are given by Professor Unwin in "The Elements of Machine Design," Part I., page 209.

Wöhler and others have shown by experiments that the safe working stress allowed in materials depends greatly on

Working Stresses in Shafting.

	Working stress.		
	Steel.	Wrought iron.	Cast iron.
Stress changing little during work, and not reversing	13,500	9,000	3,600
Part of the stress reversing at each revolution........................	9,000	6,000	2,400
Stress constantly changing between equal and opposite values	4,500	3,000	1,200

the range of stress as well as on its magnitude, for which reason we have the three divisions in the above table.

Now, if we consider a very small rectangular particle A B C D, fig. 73, forming part of the shaft, with a shearing stress f per square inch on the sides B C and A D in opposite directions, and if t be the very small thickness of the particle perpendicular to the paper, then these stresses produce a couple, f B C . t . A B. To obtain equilibrium there must therefore be shearing stresses f^1 per square inch on A B and C D, such that

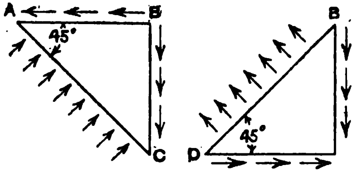


FIG. 74.

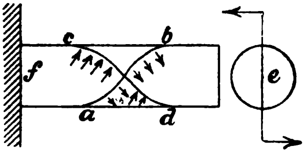


FIG. 75.

$$f^1 \mathrm{AB} . t . \mathrm{BC} = f . \mathrm{BC} . \mathrm{AB} ;$$

$$\therefore f^1 = f.$$

Shearing stress on one plane must therefore produce, or be accompanied by, an equal stress on a plane at right angles.

Next, by taking a particle, fig. 74, having A B and B C equal, and A B C a right angle, it is clear that there can be no shearing stress on A C, but that the resultant of the stress must pass through B, and is at right angles to A C. Let this stress be f_1, then

$$f_1 \, AC \, . \, t = f \, AB \, t \sqrt{2}.$$
$$f_1 = f$$

and is compressive. Similarly the stress on B D is tensile, so that a shearing stress produces two normal stresses on planes at 45 deg. to the plane of shearing, these being equal in magnitude, but opposite in sign.

If *e*, fig. 75, represents the end view of a cylindrical shaft upon which a couple acts, and which is fixed at the other end *f*, and if spirals at an inclination of 45 deg. to the axis be traced on its surface, then there will be a compressive stress perpendicular to the right-hand spirals, such as *d c*, and a tensile stress perpendicular to the left-hand spirals, as shown by the arrows; or, to look at it in another manner, if the shaft were made up of a number of spirals of wire such as *c d*, the stress in these will be tensile; but if the twisting moment were reversed, the direction of the spiral would also have to be reversed, or the wires would be unable to transmit the torsion. If the reader will twist a handkerchief, he will obtain an example of what we have proved above.

HORSE POWER TRANSMITTED BY SHAFT.

If a couple T inch-pounds makes N rotations per minute, the horse power is

$$\text{H.P.} = \frac{2\pi \, . \, T \, . \, N \, .}{12 \times 33000}.$$

$$\text{and } T = \frac{\pi}{16} f d^3$$

$$\therefore \; \text{H.P.} = \frac{2\pi^2 f d^3 \, N}{12 \times 16 \times 33000}$$

$$d^3 = 321000 \, . \, \frac{\text{H.P.}}{f \, . \, N \, .}.$$

If the twisting moment varies, the stress allowed depends on its maximum value T_1, and also on its range of variation; so that if

$$T_1 = k \, T$$

where T is the mean twisting moment,

$$d^3 = 321000 \frac{k \, . \, \text{H.P.}}{f \, . \, N}.$$

For hollow shafts replace d^3 by

$$\frac{d^4{}_1 - d^4{}_2}{d_1}.$$

SHAFTS SUBJECT TO TORSION AND BENDING OR TORSION.

We rarely find a shaft that is subject to torsion alone; in addition, we frequently find stresses caused by bending; when this is the case, the subject may be treated as follows:

Let C A B, fig. 76, represent a small triangular plate of thickness t, and let the shearing stresses on C A, A B be f, and the normal stress on A B be f_1; we shall show that by choosing a suitable value of θ the stress f_2 on C B will be wholly normal; for if D E are the middle points of C B, B A, the resultant of the shearing stresses will act along A D, and the resultant of the normal stress on A B will act along D E. Let f_3 be the shearing stress along C B; then, resolving along C B,

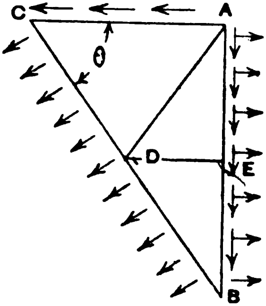


FIG 76.

$$f_3 \, CB = f \, . \, (CA \cos \theta - AB \sin \theta) - f_1 \, AB \cos \theta$$

$$f_3 = f \, (\cos^2 \theta - \sin^2 \theta) - f_1 \sin \theta \cos \theta.$$

If f_3 be zero, then

$$\frac{\tan 2\theta}{2} = \frac{f}{f_1},$$

which will give two values of θ differing by 90 deg., so that there are two planes at right angles upon which the stress is wholly normal; resolving parallel to C A,

$$f_2 \, CB \sin \theta = f_1 \, . \, AB - f \, . \, CA.$$

$$\therefore f_2 \sin \theta = f_1 \sin \theta - f \cos \theta.$$

Again, resolving parallel to A B,

$$f_2 \, \mathrm{C\,B} \cos \theta = f \, \mathrm{A\,B}.$$

$$\therefore f_2 \cos \theta = - f \sin \theta$$

and

$$(f_2 - f_1) \sin \theta = - f \cos \theta$$

$$\therefore f_2 (f_2 - f_1) = f_2$$

$$f_2 = \tfrac{1}{2} (f_1 \pm \sqrt{f_1^2 + 4 f^2}).$$

Suppose, now, that this particle is at the surface of a shaft subjected to bending and torsion, C A being parallel to the axis of the shaft, and suppose the point so chosen that f_1 is the greatest tensile stress due to bending; if, for example, the shaft is on two supports and loaded at the centre, then the lowest point of the central section will be referred to; then

$$f_1 = \frac{M}{\frac{\pi}{32} d^3}$$

where M = the bending moment

$$f = \frac{T}{\frac{\pi}{16} d^3}$$

$$\therefore f_2 \frac{\pi}{16} d^3 = M \pm \sqrt{M^2 + T^2}$$

We need only consider the greater value, and, if

$$T_e = f_2 \frac{\pi}{16} d^3$$

then

$$T_e = M + \sqrt{M^2 + T^2};$$

T_e being called the twisting moment that is equivalent to the twisting moment T and bending moment M combined, because, acting alone, it would produce the same stress. If M_e is the equivalent bending moment,

$$M_e = \tfrac{1}{2} (M + \sqrt{M^2 + T^2}).$$

These formulæ are only true for circular sections. We mention this, as we have seen them used for rectangular sections.

Next, let us suppose the shaft subject to a tensile stress f_1, so that the force P acting on it is

$$P = f_1 \frac{\pi}{4} d^2,$$

and that there is no bending.

Then
$$f_2 = \tfrac{1}{2} f_1 \pm \tfrac{1}{2}\sqrt{f_1^2 + 4f^2}$$
$$= \tfrac{1}{2}\left\{\frac{P}{\frac{\pi}{4}d^2} \pm \sqrt{\frac{P^2}{\left(\frac{\pi}{4}d^2\right)^2} + \frac{4T^2}{\left(\frac{\pi}{16}d^3\right)^2}}\right\}$$
$$= \frac{2}{\pi d^2}\left\{P \pm \sqrt{P^2 + 64\frac{T^2}{d^2}}\right\}$$

If tension, bending, and torsion act together, then
$$f_2 = \tfrac{1}{2} f_1 \pm \sqrt{f_1^2 + 4f^2}$$
and
$$f_1 = \frac{P}{\frac{\pi}{4}d^2} + \frac{M}{\frac{\pi}{32}d^3}.$$

MARINE-ENGINE SHAFTS.

We have shown above that
$$d^3 = 321000\,\frac{k\,.\,\text{H.P.}}{f\,.\,N},$$
where H.P. is the horse power, N the number of revolutions per minute, and k the ratio of maximum to mean twisting moment. It is customary to use a formula obtained from the above, although the shaft may be subject to stresses due to bending and other causes. We may write—
$$d^3 = K\,.\,\frac{\text{H.P.}}{N}$$
where K is a constant for a given type of marine engine.

In Seaton's "Manual of Marine Engineering," the following values of K are given :—

Type of engine.	K for crank.	K for tunnel.
Single-crank, two-cylinder	130	110
Two-crank, two-cylinder compound	100	85
Three-crank, three-cylinder compound	90	78
Three-crank, triple-expansion	85	74

The following table gives the diameters of crank shafts of mercantile marine engines, with corresponding values of K.

SHAFTING OF MERCANTILE MARINE ENGINES.

Diameters of cylinders.	Stroke.	Boiler press're	*d.*	I.H.P.	N.	K.	Type.
5½, 8½, 14	8	160	2⅝	30	380	85·6	T
8, 13, 21	16	160	4	147	170	74	T
14, 22, 36	24	160	6¾	907	184	62·3	T
19, 30, 49	36	160	10	1049	98	93·25	T
19½, 32½, 53	36	150	10½	750	73	112·8	T
20, 33, 54	33	150	10½	899·4	70	90·5	T
21, 35, 57	39	160	11½	909	61	102	T
25½, 42, 68	45	150	13	1672	67	88	T
27, 45, 71	48	150	13½	1752	62	87	T
34, 56, 90	60	150	17½	4000	..	..	T
34½, 57½, 92	60	170	17½	5838	80	73·4	T
44, 63, 95	45	125	18	6400	..	..	T
38, 61½, 100	66	155	18¼	..	..	..	T.
37, 61, 98½	60	160	18·9	6148	75	82·2	T
43, 66, 92	66	150	19½	7010	67¾	71·7	T.
43⅜, 67, 106¼	63	150	19½	8364	91	80·6	T
45, 71, 113	60	150	20¼	10600	89	69·5	T
37, 79, 98	69	165	26	..	..	..	T
32, 46½, 64½, 92	60	180	18	5080	70·2	80	Q.*t.*
9. 9, 18, 32..........	22	150	5¾	350	120	130·4	D.T.*t.*
18, 26, 36, 52........	39	180	11¾	1184	68½	94·1	Q.*t.*
37, 61, 71, 71........	60	160	19	3400	62	125	T.*t.*
15¾, 22, 44	33	150	8¾	618	76	82·5	semi-tandem triple.
51, 86	42	60	13½	1579	60	93·6	C
26, 48	36	80	9¼	700	88	99·5	C
36, 70	48	90	13	1536	72	103	C
76, 120	60	60	22	3430	55	170	C
57¼, 90⅛	48	60	15½	1977	60	112	C
32, 62	42	90	11	927	62	88 7	C
60, 85, 85	60	75	20	6020	85	112	C_1
63, 80, 80	60	75	21	5300	64	112	C_1
62, 90, 90	66	86	22½	6306	55	99·5	C_1
72, 100, 100	78	90	25	10350	53	79·6	C_1
Three $35\frac{7}{16}$ } Three 74⅛ }	67	60	23⅝	8006	63	103	C_1

In the table T stands for triple-expansion engine, with three cranks at 120 deg.; C for compound, with cranks at 90 deg.; D.T.*t.* for disconnective triple tandem, each pair of cylinders having separative crank and tunnel shafts; Q *t.* for quadruple tandem; T.*t.* for triple tandem; C_1 for three or six cylinder compound, with three cranks at 120. Hollow shafting is very rarely used in the mercantile marine, on account of the additional expense, but in the navies of this and other countries it is the rule. If d_1, d_2 be the external and internal diameters, and if

$$\frac{d_2}{d_1} = m,$$

Hollow Shafting.

	Boiler pressure.	d_1	d_2	I.H.P.	N	K	m	Cylinders.	Stroke.
H.M.S.....	150	9¾	5½	..	..	..	·56	27, 40¼, 60	27
U.S.N.....	160	10¾	5	2400	150	73·5	·46	25, 36, 56	36
H.M.S.....	150	11½	5¾	3963	165·3	59·5	·50	30½, 45, 68	33
H.M S.....	135	12	5	3000	135	75	·42	30, 44, 68	36
U.S.N.	160	13	4	4500	132	63·75	·31	35½, 57, 88	36
U.S.N.	160	13½	6	4000	129	76	·44	32, 46, 70	42
U.S.N.....	160	14	6	4500	129	76	·43	34½, 48, 75	42
U.S.N.....	160	16	7½	6750	129	74·5	·47	42, 59, 92	42
H.M.S.....	135	16	8	6000	95	60·7	·5	43, 62, 96	51
U.S.N. ...	160	17	7½	8000	129	76	·44	*	*
U.S.N.	160	9	3	1500	150	72	·33	19¾, 30¾, 52¾	30
U.S.N.....	160	11	4	2700	150	72·6	·363	27, 41, 64	30
U.S.N.	160	13½	6	5000	164	72	·44	36, 53, 57, 57	33
U.S.N.	160	10	5	2700	185	64·2	·5	26½, 39, 63	26
U.S.N.	160	12½	6	3750	150	74·1	·48	31¼, 46, 70	36
U.S.N.	160	7	3½	800	200	80·4	·5	15¾, 22½, 35	24
U.S.N.	160	6	3	650	240	75	·5	13½, 21, 31	20

---

* This shaft transmits the power of two sets of engines, each having cylinders 32 in., 46 in., 70 in., and 42 in. stroke.

then $$\frac{d_1{}^4 - d_2{}^4}{d_1} = d_1{}^3 (1 - m^4)$$

and $$d_1{}^2 - d_2{}^2 = d_1{}^2 (1 - m^2);$$

so that if $$m = \tfrac{1}{2}$$

the reduction in weight is 25 per cent, and in strength is only 6¼ per cent; weight for weight, then, a hollow shaft is stronger than a solid. For the navy hollow shafting is made of fluid-compressed steel. Its additional strength may be judged from the values of K in the foregoing table, in which H.M.S. stands for the English navy, and U.S.N. for that of the United States. The engines are in all cases, except one with four cranks, triple-expansion, with three cranks at 120 deg., and

$$\mathrm{K} = \frac{\mathrm{N}\,(d_1{}^4 - d_2{}^4)}{d_1 \times \mathrm{H\,P.}}.$$

LLOYD'S RULES FOR CRANK SHAFTS (1892).

For compound engines with two cranks at right angles—

$$d = (\cdot 04\,\mathrm{A} + \cdot 006\,\mathrm{D} + \cdot 02\,\mathrm{S}) \times \sqrt[3]{\mathrm{P}}.$$

For triple-expansion engines with three cranks at equal angles—

$$d = (\cdot 038\,\mathrm{A} + \cdot 009\,\mathrm{B} + \cdot 002\,\mathrm{D} + \cdot 0165\,\mathrm{S}) \times \sqrt[3]{\mathrm{P}}.$$

For quadruple-expansion engines with two cranks at right angles—

$$d = (\cdot 034\,\mathrm{A} + \cdot 011\,\mathrm{B} + \cdot 004\,\mathrm{C} + \cdot 0014\,\mathrm{D} + \cdot 016\,\mathrm{S} \times \sqrt[3]{\mathrm{P}}.$$

For quadruple-expansion engines with three cranks—

$$d = (\cdot 028\,\mathrm{A} + \cdot 014\,\mathrm{B} + \cdot 006\,\mathrm{C} \times \cdot 0017\,\mathrm{D} + \cdot 015\,\mathrm{S}) \times \sqrt[3]{\mathrm{P}}.$$

For quadruple-expansion engines with four cranks—

$$d = (\cdot 033\,\mathrm{A} + \cdot 01\,\mathrm{B} + \cdot 004\,\mathrm{C} + \cdot 0013\,\mathrm{D} + \cdot 0155\,\mathrm{S}) \times \sqrt[3]{\mathrm{P}}.$$

Where A = diameter of high-pressure cylinder in inches.
B = diameter of first intermediate in inches.
C = diameter of second intermediate in inches.
D = diameter of low-pressure cylinder in inches.
S = stroke in inches.
P = boiler pressure above atmosphere in pounds per square inch.

The screw shaft is to be the same diameter as is required for the crank shaft.

Intermediate shafting should be at least ·95 of the diameter of the crank shaft.

NOTE.—The rules are intended to apply to two-cylinder compound engines, in which the ratio of areas of low to high pressure cylinders does not exceed 4·5 to 1; for triple-expansion engines when the ratio does not exceed 9 to 1; for quadruple-expansion engines when the ratio does not exceed 12 to 1; and when the length of stroke is not less than half the diameter, or not greater than the diameter of the low-pressure cylinder.

BOARD OF TRADE RULES FOR SHAFTS.

Diameter of shaft.

$$= \sqrt[3]{\frac{C.P D^2}{f\left(2 + \frac{D^2}{d^2}\right)}}$$

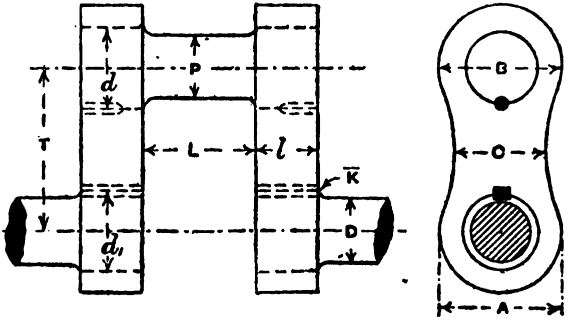


FIG. 77.

C = half the stroke in inches.
P = absolute pressure in the boiler.
d^2 = square of diameter of high-pressure cylinder in inches, or sum of squares of diameters if there are several.
D^2 = a similar quantity for the low-pressure cylinder or cylinders.
f = a constant from the following table.

When there is only one crank the constants applicable are those opposite 180 deg., and for paddle engines the above constants are multiplied by 1·4. In most cases it will be found that shafts are larger than those obtained by the above rules.

In the mercantile marine, built-up crank shafts are generally used in preference to solid shafts, as, although heavier, they are more reliable. The faults of cranks forged in one piece are weakness at the junction of shaft or pin and web, flaws in the web, and scarf ends in the pin.* One form of built-up crank is shown in fig. 77. The webs are held to the parts of the shaft by pins or keys, the former being more usual. The crank pin is also held to the web by a small pin at each end. The latter are usually smaller than the former, although there is a considerable amount of torsion on the after crank pin. The methods of shrinking the five parts together are as follow: A sort of Bunsen burner, fig. 78, supplied with gas and air, is placed in one eye of a web, and when the heat has sufficiently expanded it the large end of the shaft is placed in the hole; after cooling, the hole for the pin is drilled, and it is driven in by a hammer. If key-ways have been cut in shaft and web,

For two cranks. Angle between cranks.	For crank and propeller shafts.	For tunnel shafts.
Degs.	*f*.	*f*.
90	1,047	1,221
100	966	1,128
110	904	1,055
120	855	997
130	817	953
140	788	919
150	766	894
160	751	877
170	743	867
180	740	864
For three cranks.		
120	1,110	1,295

---

* See a paper on the "Forging of Crank Shafts" in the Proceedings of the Mechanical Engineers, 1879.

care must be taken to bring the two together. When both webs have been treated in this way, the two parts of the shaft are bolted on V blocks and carefully aligned; the eyes for the crank pin are next got in line, and are heated together, and the pin is then drawn through until it is in place. Another method is to shrink shafts and webs together as before; the pin is next shrunk into one web, and the three parts are then bolted down on a table. The remaining eye of the other web is next heated and placed on the other end of the pin, and the web is pulled round until it touches the table. As both webs have been slotted to exactly the same size, and as the first part of the shaft and the pin are bolted down with their axes horizontal, the web being slightly raised at the pin end and touching the table at the shaft end, the two parts of the shaft are thus brought into line. The pin is finished before shrinking, the shaft afterwards. The table on next page gives dimensions of these cranks from practice.

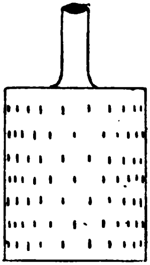


FIG. 78.

The following rules give proportional dimensions:—

$l = \cdot 63$ to $\cdot 77$ D.
$d = 1\cdot 05$ to $10\cdot 6\,D = d_1$.
$P = D$, occasionally a little more.
$A = 1\cdot 85$ to $2\cdot 08$ D.
$B = 1\cdot 76$ to $1\cdot 94$ D.
$C = 1\cdot 3$ to $1\cdot 68$ D.

Foster's crank shaft is in three parts, two of which form the shaft, and the third the pin and webs. This last is of mild cast steel; the ends of each part of the shaft are enlarged, and the three parts are shrunk together. Keys are also used, as in fig. 77.

Dickinson's crank is also in three parts, similar to the above, excepting that the webs are bolted to flanges formed on the two parts of the shaft, these flanges being sunk into the webs.

The length L of the crank pin may be calculated as follows: Let p be the boiler pressure and d_2 the diameter of the H.P. cylinder; then

$$L = \frac{p\,d^2 \times \cdot 7854}{p_1 \cdot P}$$

where p_1 is from 400 to 600, and may be taken as 500 with advantage, so that the pressure per square inch on the pin is p_1.

D.	P.	d_1.	d.	l.	A.	B.	C.	K.	T.
9⅞	9⅞	10⅜	10⅛	6¼	19⅜	19⅛	15	2¼ pin	18
11½	11½	12¼	12¼	7½	21¼	20¼	15¼	3 × 2	21
12¼	12¾	12½	13	8	24	24½	19½	2½ pin	21
12½	12½	13¼	12½	8⅝	26	22½	17	..	24
12½	13	13	13	8⅝	25	25	19¾	2½ pin	22½
13	13	13¾	13¾	10	25	24	19	2½ × 1¾	24
14	14½	14½	14½	10	26	26	20	2½ pin	24
14¾	15	15	15	11	29½	29½	29½	2½ pin	27
15	15	15½	15	10	29	29	29	2½ pin	24
16½	17	17	17	12	32	32	32	2½ pin	30
18	18	19¼	18	12½	31	31	25½	..	30
18·9	20·5	20	20·5	13·8	39	39	31·4	..	30
19½	20½	20⅝	20½	14	40	40	33½	..	31½
23	23	24½	24	17	43½	42	31	6 × 4	36
25*	26*	27*	28*	18	45	45	45	..	36

* Hollow; internal diameters 14, 12, 12, 12 respectively.

Fownes' patent is shown in fig. 79, the dotted portion of the right-hand figure showing the quantity of web saved. The parts of the pin shrunk into the webs are eccentric to the central portion. It is said to occupy no more space than the ordinary solid shaft.

NUMERICAL EXAMPLES ON SHAFTING.

Example I.—A hollow steel shaft replaces a solid shaft of wrought iron of the same diameter. The stress that the steel can safely bear is 35 per cent more than the stress in the wrought-iron shaft. They have the same strength to resist torsion. Find the internal diameter of the former, and the saving of weight, neglecting the couplings.

Let $m = \frac{d_2}{d_1}$ where $d_1\ d_2$ are the external and internal diameters. Since they are of equal strength,

$$f d_1^3 = f_1 \frac{d_1^4 - d_2^4}{d_1}$$

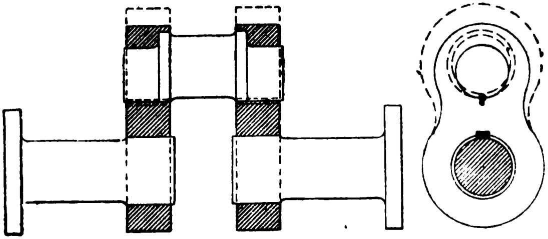


FIG. 79.

where f, f_1 are the stresses in the wrought-iron and steel shafts. But $f_1 = 1{\cdot}35 f$.

$$\therefore d_1^3 = 1{\cdot}35\, d_1^3 (1 - m^4)$$

$$m^4 = 1 - \frac{1}{1{\cdot}35}$$

$$m = {\cdot}71.$$

The percentage saving of weight is

$$100 \frac{d_2^2}{d_1^2} = 50.$$

Example II.—Compare the weight of shafting in a twin with that in a single-screw ship, neglecting the couplings. The lengths of shafts and horse powers transmitted are the same. The number of revolutions of each of the shafts in the twin-screw ship is $1\frac{1}{4}$ that of the single screw, and the ratios of maximum to mean twisting moment of all three engines are equal.

Let D = diameter of single shaft

d = diameters of twins

$$\text{Then } D = \sqrt[3]{\frac{K.\ H.P.}{N.}}$$

$$d = \sqrt[3]{\frac{K.\ H.P.}{1{\cdot}25 \times 2\,N.}} = \sqrt[3]{\frac{K.\ H.P.}{2{\cdot}5\,N.}}$$

where N = the number of revolutions per minute of single shaft.

$$\frac{D^2}{d^2} = \left(2{\cdot}5\right)^{\frac{2}{3}}$$

$$\therefore \frac{2d^2}{D^2} = 1{\cdot}095,$$

which is the ratio required.

Example III.—The outline of an overhung crank is shown in fig. 80, and P acts at right angles to the plane of the paper. Find the diameter of the shaft at the journal if P is 40,000 lb. and the stress allowed is 5,000 lb.

It is usual, although not strictly correct, to take the reaction of the journal as concentrated at its centre; assuming this to be the case,

$$M = 16\ P \text{ inch-pounds}$$

$$T = 18\ P \text{ inch-pounds}$$

$$\therefore T_e = P\left\{16 + \sqrt{16^2 + 18^2}\right\}$$

$$\text{and } \frac{\pi}{16} f\,d^3 = 40000 \times 40{\cdot}1$$

$$d^3 = 1630$$

$$d = 11{\cdot}8 \text{ inches nearly.}$$

In the table on page 98 we have worked out the stresses in the journals of overhung cranks, as in Example III. It gives a somewhat larger value than that which would be obtained by supposing the reaction of the journal uniformly distributed, but the difference is not so great as one might imagine. In one case, in which the stress is 7,820 lb. in the table, it is only reduced to 7.120 lb. by taking into account the length of the journal. We cannot even be certain that the reaction of the journal is uniform, and its resultant may be on the crank side of its centre. If we lengthen the journal, the above theory would increase the stress, but it is absurd to suppose this to be the case. In the table, A is the distance between the centre of the journal and the plane in which the centre of the pin rotates, and B is the half-stroke.

Example IV.—In a marine engine the distance from centre to centre of bearings is l in., the length of stroke $2r$ in., the twisting moment T inch-pounds, and the force on low-pressure crank pin R lb. The acute angle between the direction of the connection rod and the crank is θ. Find the stresses f_1, f_2 at the aft journal and crank pin, each part of the shaft being treated as an independent beam.

The subject of beams on several supports is one of great difficulty, and for this reason the last supposition has been made; and as it over-estimates the stress in the crank pin, which is greater than that in the journal, it is on the safe side. We shall also assume that the reactions of the bearings are concentrated at their centres, and that the force R is at the centre of the pin. The couple exerted by the bearings is also unknown; let this be

$$M = n R l.$$

If the bearings can be treated as supports only, then $n = 0$; but they probably exert some bending moment on the shaft,

Boiler pressure.	Diameter of cylinder.	Shaft diameter.	A.	B.	*f.*	Type.
32*c.*	22¼	7¾*s.*	15½	20	7,620	H.S.
60*c.*	27	9½*w.*	18½	18	10,850	H.S.
80*c.*	14½	6½*s.*	11¼	15	8,430	H.S.
60*n.*	9	8⅝*s.*	8	9	8,420	H.S.
60*n.*	10	4¼*w.*	9½	10	7,280	H.S.
80*n.*	42	18*s.*	30	42	6,740	H.S.
100*n.*	11·81	5⅛*w.*	11	11·81	11,250	H.S.
80*c.*	40	18*s.*	31½	60	10,000	H.S.
80*c.*	14	7*s.*	15	15	6,600	H.S.
100*c.*	32	17*c.s.*	37½	42	8,820	H.S.

S., for steel; *w.*, for wrought iron; *c.s.*, for fluid compressed steel.

whose magnitude is unknown. If the shaft be treated as a *uniform* beam, loaded at the centre, then $n = \frac{1}{8}$. This probably over-estimates the bending moment at the journal, which would be $\frac{Rl}{8}$; and under-estimates that at the pin, which is greatest when n is zero. For the journal we shall assume $n = \frac{1}{8}$, and for the pin that $n = 0$, which is on the safe side, as it over-estimates the stress in each case.

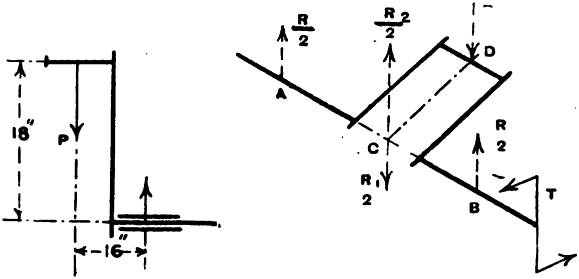
At the journal whose diameter is d_1,

$$T_e = f_1 \frac{\pi}{16} d_1^3 = \frac{Rl}{8} + \sqrt{\frac{R^2 l^2}{64} + T^2}$$

$$\therefore f_1 = \frac{\cdot 636\,(Rl + \sqrt{R^2 l^2 + 64\,T^2})}{d_1^3}.$$

To find f_2, let two equal and opposite forces $\frac{R_1}{2}$ and $\frac{R_2}{2}$ (fig. 81) each equal and parallel to $\frac{R}{2}$ be placed at C, the centre of AB, and let them be supposed rigidly connected to the crank; this cannot alter the stress at any point. Then $\frac{R}{2}$ at B and $\frac{R_1}{2}$ form a couple causing bending at the centre of the pin, whose magnitude is

$$M_1 = \frac{Rl}{4}$$



FIGS. 80 and 81.

which is the bending moment at the centre of the pin because $n = 0$. The twisting moment at D is

$$T_1 = T - \frac{R_2\, r \sin\theta}{2}$$

$$= T - \frac{1}{2} T_2$$

where T_2 is the twisting moment of the low-pressure crank alone.

$$f_2 \frac{\pi}{16} d_2^3 = M_1 + \sqrt{M_1 + T_1^2}$$

which gives f_2.

Example V.—In a compound locomotive there are two high-pressure cylinders and one low-pressure. The latter is connected to a single-throw crank, whose dimensions are shown in fig. 82. The greatest pressure on the low-pressure piston is 50,000 lb., and the weight on each journal 17,000 lb. The diameter of each wheel is 85 in., of the journal 7 in., and of the pin $7\frac{3}{4}$ in. It is assumed that equal power is transmitted to each wheel. A, B are the journal centres, and C, D the points at which the resultants of the forces exerted by the wheels on the shaft act. To find the greatest possible stresses in the journal and crank pin, the obliquity of the connecting rod may be neglected.

It is evident that the greatest stresses occur when the crank is at right angles to the line of stroke, but they are not the same when the pin is above the line of stroke as

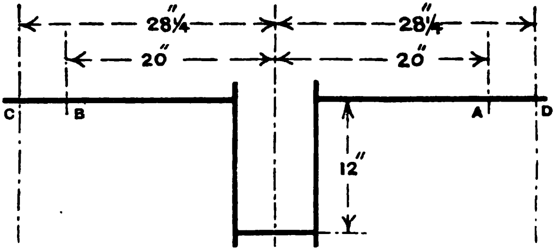


Fig. 82.

when it is below. In either case, let F be the force at the point of contact of rail and driving wheel; then

$$42\tfrac{1}{2}\,F = 25000 \times 12$$
$$F = 7050 \text{ lb.}$$

This force at the rail is equivalent to a force of 7,050 lb. at the wheel seat, and a couple whose moment is 300,000 inch-pounds. Fig. 83 is the diagram of forces when the crank is above the line of stroke. The downward forces at A and B are balanced by upward forces at D and C, and the pressures of 50,000 lb. on the pin, and 7,050 lb. on each wheel seat, are balanced by 32,050 lb. acting horizontally at the journals and the two couples at the end of the shaft.

The bending moment at the journal is

$$M = AD \times \sqrt{17000^2 + 7050^2}$$
$$= 152000 \text{ inch-pounds, nearly.}$$

Also
$$T = 300000 \text{ inch-pounds}$$
$$T^e = M + \sqrt{M^2 + T^2}$$
$$= 489000 \text{ inch-pounds}$$

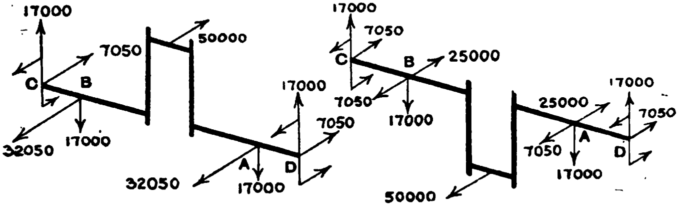
and
$$d = 7 \text{ in. at the journal;}$$
$$\therefore f_1 = \frac{16}{\pi} \times \frac{489000}{7^3} = 7260 \text{ lb.}$$

The stress at the pin is only caused by bending, for if we take moments about the centre line of the pin, considering, of course, only the right-hand half of the shaft, we get

$$T_1 = 7050 \times 12 - 32050 \times 12 + 300000$$
$$= 0.$$

The horizontal bending moment at the centre of the pin is

$$M_1 = 32050 \times 20 - 7050 \times 28\tfrac{1}{4}$$
$$= 442000 \text{ inch-pounds.}$$



Figs. 83 and 84.

The bending moment in a vertical plane is

$$M_2 = 17000 \times 8\tfrac{1}{4} = 140000 \text{ inch-pounds.}$$

The total bending moment is

$$\sqrt{M_1^2 + M_2^2} = 454000 \text{ inch-pounds.}$$
$$f = \frac{32}{\pi} \times \frac{454000}{(7\tfrac{3}{4})^3} = 9950 \text{ lb.}$$

Fig. 84 is the diagram of forces when the crank is below the line of stroke. The moments acting on the journal, and therefore the stresses, are the same as before. On the pin the twisting moment is zero. The horizontal bending moment is

$$\begin{aligned} M_1 &= 7050 \times 8\tfrac{1}{4} + 24000 \times 20 \\ &= 558150 \text{ inch-pounds.} \end{aligned}$$

The vertical bending moment is

$$\begin{aligned} M_2 &= 17000 \times 8\tfrac{1}{4} \\ &= 140000 \text{ inch-pounds} \\ \sqrt{M_1{}^2 + M_2{}^2} &= 575000 \text{ inch-pounds} \\ f_3 &= \frac{32}{\pi} \times \frac{575000}{(7\frac{3}{4})^3} \\ &= 12600 \text{ lb.} \end{aligned}$$

This example is taken from Webb's "Compound Locomotive." The diameter of the low-pressure cylinder is 30 in., and the relief valve for the low-pressure steam chest is loaded to 80 lb. per square inch; the greatest possible load on the low-pressure piston is therefore 56,500 lb., nearly, somewhat more than we have taken above. The load carried by the corresponding driving wheels is 15 tons 10 cwt., giving 17,360 lb. on each journal. Drawings may be found in *Engineering*, July 25th, 1890. Indicator diagrams of this type give a pressure of about 40 lb. per square inch on the low-pressure piston.

Example VI.—In a locomotive with inside cylinders the pressure on each piston is 38,000 lb. There are no coupling rods, and the diameter of each driving wheel is 7 ft. Find the greatest stresses f_1, f_2, f_3 at journals, middle of shaft, and pins. The diameter of journal is $7\frac{1}{2}$ in., of middle of shaft 7 in., and of the pin 8 in. The vertical load on each journal is 17,000 lb. Dimensions are as follow: Centres of cylinders 24 in., centres of journals 46 in., gauge of rails $56\frac{1}{2}$ in.

If one of the cranks is inclined at an angle θ to the line of stroke, the twisting moment is $Pr(\sin\theta + \cos\theta)$ for values of θ between 0 deg. and 90 deg., and $Pr(\sin\theta - \cos\theta)$ for values between 90 deg. and 180 deg., P being the force on each pin, and r the half stroke. The moment of each wheel on the shaft is $\frac{1}{2}Pr(\sin\theta \pm \cos\theta)$. The moment transmitted by the middle of the shaft is therefore

$$\begin{aligned} \tfrac{1}{2}Pr(\sin\theta \pm \cos\theta) &\mp Pr\cos\theta \\ = \tfrac{1}{2}Pr\sin\theta &\mp \cos\theta \end{aligned}$$

where the negative sign applies to values of θ between 0 deg. and 90 deg., and the positive to values between 90 deg. and 180 deg. This is the same as

$$\tfrac{1}{2}\,P\,r\,(\sin\theta - \cos\theta)$$

where θ has any value between θ deg. and 90 deg. Evidently this will be the greatest when θ is 90 deg., and it is also clear that the bending moment is greater when the forces on the pins are in the same direction than when they are in opposite directions.

Fig. 85 shows one crank vertical and the other horizontal. This is the position of greatest bending, for if A B are the wheel seats, E D the journals, and E *any point between the centres of the two pins*, the horizontal moment at the point is

$$M_1 = F \times BD + P \times DG.$$

If the crank is above the line of stroke, then

$$M_1 = P \times DG - F \times BD$$

less than the above.

$$F = \frac{P\,.\,r}{2\,R}$$

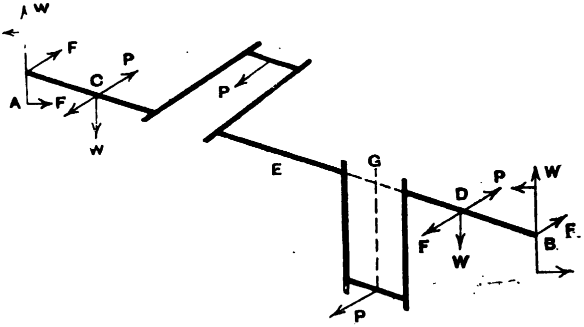


FIG. 85.

where F is the force between rail and driving wheel, 2 R is the diameter of the wheel,

$$= \frac{38000 \times 12}{84} = 5430 \text{ lb.}$$

and the couple at each end of the shaft is

$$T = 19000 \times 12 = 228000 \text{ inch-pounds}$$

$$M_1 = 5430 \times 5\tfrac{1}{4} + 38000 \times 11$$

$$= 446500 \text{ inch-pounds.}$$

The vertical moment is

$$M_2 = W \times BD = 17000 \times 5\tfrac{1}{4} = 89250 \text{ inch-pounds.}$$

The resultant bending moment is

$$M = \sqrt{M_1^2 + M_2^2} = 455000 \text{ inch-pounds.}$$

The twisting moment is T;

$$\therefore T_e = M + \sqrt{M^2 + T^2}$$

$$= 964000 \text{ inch-pounds.}$$

This is the equivalent twisting moment at centre of shaft and pins;

$$\therefore f_2 = \frac{16}{\pi} \times \frac{964000}{7^3} = 14300 \text{ lb.}$$

and $$f_3 = \frac{16}{\pi} \times \frac{964000}{8^3} = 9600 \text{ lb.}$$

The greatesst stress in the journals is when both cranks are at 45 deg. to the vertical. The total twisting moment is then

$$Pr\sqrt{2};$$

and $$F = \frac{P.r.\sqrt{2.}}{2R.} = 7650 \text{ lb.}$$

The bending moment on the journal is

$$M_3 = 5\tfrac{1}{4}\sqrt{7650^2 + 17000^2}$$

$$= 90149 \text{ inch-pounds.}$$

The twisting moment of each wheel is

$$T_3 = \frac{Pr}{\sqrt{2}} = \frac{38000 \times 12}{\sqrt{2}} = 323000 \text{ inch-pounds.}$$

$$\therefore T = M_3 \times \sqrt{M_3^2 = T_3^2}$$

$$= 426000.$$

$$f_1 = \frac{16}{\pi} \cdot \frac{426000}{(7\frac{1}{2})^3} = 5150 \text{ lb.}$$

Example VII.—Three engines of equal power drive one crank shaft, and the resisting moment is applied to one end only of the shaft. The shaft is made in three similar forgings, which are coupled together by bolts through forged flanges. Each length of the shaft is mounted on two bearings, and each crank pin is midway between the centres of the bearings. Show that the mean twisting moment on the crank pin nearest the end to which the twisting moment is applied is $23·5 \frac{\text{H.P.}}{\text{N}}$ inch-tons. If θ be the acute angle between crank and direction of connecting rod, and R be the pressure on the rod, the twisting moment of that crank alone is $\text{R} . r . \sin \theta$, r being the length of the crank in inches. If T be the twisting moment of all three engines, fig. 81, that on the after crank pin is

$$\text{T}_1 = \text{T} - \tfrac{1}{2} \text{R} r \sin \theta.$$

But the mean value of $\text{R} r \sin \theta$ is

$$\text{T}_2 = \frac{\frac{1}{3} \text{H P.} \times 33000 \times 12}{2 \pi \text{N} \times 2240} \text{ inch-tons,}$$

and
$$\text{T}_3 = \frac{\text{H.P.} \times 33000 \times 12}{2 \pi \text{N} \times 2240} \text{ inch-tons,}$$

where T_3 is the mean value of T.

If, then, T_4 be the mean value of T_1,

$$\text{T}_4 = \tfrac{5}{6} \times \frac{\text{H.P.} \times 33000 \times 12}{2 \pi \text{N} \times 2240} = 23·5 \frac{\text{H.P.}}{\text{N}} \text{ inch-tons.}$$

Example VIII.—In the above question suppose the distance from centre to centre of the two bearings for each length of the shaft is 1¼ the stroke, and each shaft is treated for bending as if independent of the others; show that the twisting moment on the above-mentioned pin, which is equivalent to the combined mean bending and twisting moments to which it is subjected, is greater than the total mean twisting moment of the three engines, and find its value.

We shall treat the crank for bending as if the bearings are merely supports at their centres, and exert no bending moment on the shaft. Let P be the mean whole pressure on the piston in tons; then, neglecting the obliquity of the connecting rod, which would make the question exceedingly complicated, the mean bending moment is

$$\begin{aligned} \text{M} &= \frac{\text{P} \times \text{span}}{4} \text{ inch-tons.} \\ &= \tfrac{5}{8} \text{P} . r. \end{aligned}$$

But $$12\,P\,r\,N = \frac{12\ \text{H.P.} \times 33000}{2240}$$

$$\therefore\ M = \tfrac{5}{8}\,P\,r = \frac{5\ \text{H.P.} \times 33000}{8 \times 2240 \times N}$$

$$= 9{\cdot}2\,\frac{\text{H.P.}}{N}\ \text{inch-tons},$$

and $$T_4 = 23{\cdot}5\,\frac{\text{H.P.}}{N}.$$

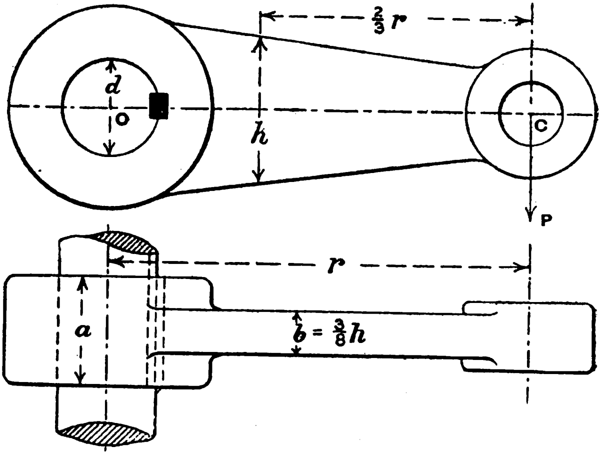


Fig. 86.

$$\therefore\ T_e = M + \sqrt{M^2 + T_4^2}$$

$$= \frac{\text{H.P.}}{N}\left(9{\cdot}2 + \sqrt{552{\cdot}5 + 84{\cdot}6}\right)$$

$$= 34{\cdot}4\,\frac{\text{H.P.}}{N}.$$

The mean twisting moment on the shaft is

$$T_3 = 28{\cdot}2\,\frac{\text{H.P.}}{N}.$$

$$\therefore\ \frac{T_e}{T_3} = 1{\cdot}22.$$

If we give M half the above value, treating the shaft for bending as if it were a uniform beam fixed horizontally at both ends, then

$$T_e = 28{\cdot}5 \frac{\text{H.P.}}{N}$$

and $$\frac{T_e}{T_3} = 1{\cdot}01.$$

Example IX.—The crank, fig. 86, is acted on by a force P applied at the pin in the central plane perpendicular to O C. It causes a bending moment on the shaft equal in magnitude to half the twisting moment produced; the crank is secured to the shaft by means of a sunk key. Determine the dimensions marked a and h in terms of d, in order that the crank-arm key and shaft may be equally strong, allowing the stress in the material to be the same for each part. The breadth of the key is $\frac{1}{4} d$.

The twisting moment is $P r$; hence

$$\begin{aligned} T_e &= M + \sqrt{M^2 + T^2} \\ &= P r \left(\tfrac{1}{2} + \tfrac{1}{2}\sqrt{5}\right) \\ &= 1{\cdot}61\, P r \\ \therefore f \frac{\pi}{16} d^3 &= 1{\cdot}61\, P r. \end{aligned}$$

This neglects the weakening of the shaft by the keyway, the effect of which has not been reduced to a mathematical formula. Considering the strength of the key, we get

$$P r = f \frac{a d}{4} \times \frac{d}{2}$$

because the area in shear is $\frac{a d}{4}$, and the distance from the centre of the shaft is $\frac{d}{2}$.

$$f \frac{a d^2}{8} = \frac{f \pi d^3}{16 \times 1{\cdot}61}.$$

$$\therefore a = {\cdot}975\, d.$$

Next for h, $$\tfrac{2}{3} P r = \frac{f}{6} b h^2,$$

and $$b = \tfrac{3}{8} h;$$

$$\therefore P r = \tfrac{3}{32} f h^3 = \frac{f \pi d^3}{16 \times 1{\cdot}61}$$

$$h = 1{\cdot}09\, d.$$

Example X.—Fig. 87 represents the crank shaft of a horizontal engine, the pressure P, which is 70,000 lb., acting at right angles to the crank, which is vertical. The journals are at A and B, and at C is carried a rope flywheel, weighing 122,220 lb., and having a diameter of 30 ft. The dimensions AB, AC, CB, AD, DB are 127·5, 70·5, 57, 31·5, and 159 respectively, D being the point on the shaft centre line produced directly below the centre of the crank pin. The directions of the ropes are approximately horizontal, and they lie in the direction of the arrow marked $F_2 + F_1$, where F_2 is the tension on the driving side, and F_1 that on the slack, the ratio, $F_2 : F_1$, being 9. The horse power at 42 revolutions per minute is 900. The journal A is 18 in. diameter, and the diameter at the flywheel is 21 in. The weakening effect of six flats for keys is not to be considered. It is required to find the stresses f_1 at the journal A, and f_2 in the shaft at the flywheel.

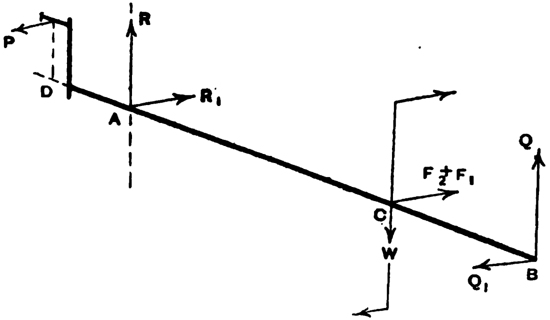


FIG. 87.

If D, N are the diameter in feet and number of revolutions per minute of the flywheel,

$$(F_2 - F_1)\pi D N = \text{H.P.} \times 33000;$$

$$\therefore F_2 - F_1 = \frac{33000 \times 900}{30\pi \times 42} = 7500 \text{ lb.},$$

and since

$$F_1 = \frac{F_2}{9} = \frac{F_2 - F_1}{8}$$

$$\therefore F_2 + F_1 = 7500 \times 1\tfrac{1}{4} = 9375 \text{ lb.}$$

The twisting moment of the crank is therefore opposed by two moments, the one caused by the resistance of the ropes and the other by the inertia of the flywheel, whose speed at this instant is increasing because the engine is supplying more power than is required. The shaft may be therefore treated as if at rest with the vertical force Q R, the horizontal forces P, Q_1, R_1, and $F_2 + F_1$, and a couple at C whose moment is equal and opposite to that of P about B D.

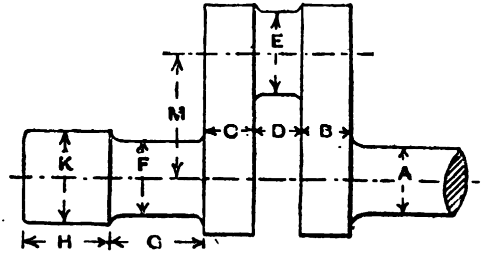


FIG. 88.

Taking moments about B for horizontal forces—

$$R_1 = \frac{P.DB - (F_2 + F_1).CB}{AB}$$

$$= \frac{70000 \times 159 - 9375 \times 57}{127{\cdot}5}$$

$$= 83100 \text{ lb.}$$

$$Q_1 = R_1 + F_2 + F_1 - P = 22400 \text{ in round numbers.}$$

$$R = \frac{W.CB}{AB} = \frac{122200 \times 57}{127{\cdot}5} = 54600.$$

$$Q = W - R = 67600.$$

The twisting moment at C is

$$T = 4200000 \text{ inch-pounds.}$$

The bending moment at C is

$$M = BC\sqrt{Q^2 + Q_1{}^2} = 4040000 \text{ inch-pounds.}$$

$$T_e = M + \sqrt{M^2 + T^2} = 9860000 \text{ inch-pounds.}$$

$$f_2 = \frac{16}{\pi}\frac{T_e}{21^3} = 5420.$$

At the crank journal

$$f_1 = 6060 \text{ lb.,}$$

which may be found as in Example III.

Fig. 88 is the crank of a locomotive. In the following table dimensions are given from actual practice. L is the depth of the webs, and N the diameter of piston. The ends of the webs are sometimes struck with radii, whose centres are the shaft and pin, in the usual manner for ordinary shafting, but more often they are semi-circular; they are generally hooped, the hoops being shrunk on as a safeguard against accident, in case a crank should break. The webs of Mr. Worsdell's crank are circular, so that they can be finished in the lathe; they are not hooped.

DIMENSIONS OF LOCOMOTIVE CRANK AXLES.

A 5¼	7	6½	6½	7	7	7	7	7	7	7½
B 4½	4⅜	4¼	4	4¾	4¼	4¾	4¼	4¼	4½	4¾
C 4⅛	4⅜	4	4	4¼	4¼	4¾	4	4	3¾	4¾
D 4	4½	4	4	4½	4	4½	4	4	4	5
E 6¼	7¼	6½	7	8	7¾	8	7½	8	7½	8
F 5¾	7	6½	7	7½	7½	7½	7½	7½	7¼	8
G 8	9	8½	7¼	9	7½	9	8½	7	8	10
H 6⅛	8½	5⅞	7½	8⅜	7½	8½	7⅛	7¼	7⅝	6¾
K 5¾	7½	7½	7¾	9	9	9	8½	8½	8¼	9
L 8	10	10	11	12	12	12	13	13½	12	13
M 10	11	10	12	12	13	12	13	13	13	12
N 13	16	16½	17	17½	17½	18	18	18¼	19	20
Boiler press ..	..	..	150	140	140	150	150	150	150	..
p 800	920	1230	1210	936	1080	1060	1270	1225	1410	1175

In the table p is the pressure per square inch on the crank pin when the full boiler pressure acts on the piston. In the cases in which we have not been able to obtain the boiler pressure we have assumed it to be 150. Here again the reader may note how high are stresses and pressures per square inch on bearing surface per locomotive practice, compared with their values in marine practice.

Example I.—In a single-cylinder engine the steel crank shaft, of 8 in. diameter, is subjected to a twisting moment, which fluctuates from 2½ times the mean twisting moment to zero. In a multi-cylinder engine, working at the same power and speed, the maximum twisting moment is 20 per cent greater and the minimum 20 per cent less than the mean. What diameter of shaft will be required in order that it may be capable of enduring the straining action to which it is subjected as the shaft of the single-cylinder engine, supposing the material to be the same? (Science and Art Honours, Machine Construction Examination, 1895.) This is evidently intended as a problem on Wöhler's researches, which have been reduced to the following mathematical equation by Professor Unwin :

$$k_{\max} = \frac{\Delta}{2} + \sqrt{K^2 - 1{\cdot}5\,\Delta\,K},$$

where $k_{\max}$ and $k_{\min}$ are the greatest and least stresses to which the material is subjected,

$$\Delta = k_{\max} - k_{\min}$$

$$K = \text{greatest safe dead load.}$$

In the former case $k_{\min} = o$, and $k_{\max}$ may be calculated from the equation—

$$2\tfrac{1}{2}\,T = k_{\max}\,\frac{\pi}{16}\,d^3$$

where T is the mean twisting moment which the engine exerts, and $d = 8$ in.

In the second case,

$$1{\cdot}2\,T = k^1_{\max}\,\frac{\pi}{16}\,d_1{}^3$$

$$\cdot 8\,T = k^1_{\min}\,\frac{\pi}{16}\,d_1{}^3$$

where $k^1_{\max}$ and $k^1_{\min}$ are the maximum and minimum stresses to which the second shaft is subjected ;

$$\therefore\ \cdot 4\,T = \Delta^1\,.\,\frac{\pi}{16}\,d_1{}^3$$

$$\therefore\ k_{\max} = \frac{k_{\max}}{2} + \sqrt{K^2 - 1{\cdot}5\,k_{\max}\,K}$$

$$\therefore\ \frac{k^2_{\max}}{4} = K^2 - 1{\cdot}5\,K\,.\,k_{\max}$$

$$k_{\max} = \cdot 6\,K.$$

In the second case,

$$\frac{\Delta^1}{k^1_{\max}} = \frac{\cdot 4\,T}{1{\cdot}2\,T}\,. = \tfrac{1}{3}\,;$$

hence

$$k^1_{\max} = \tfrac{1}{3}\,k^1_{\max} + \sqrt{K^2 - \frac{1\cdot5}{3}\,k^1_{\max}\,K}$$

$$= \tfrac{1}{3}\,k^1_{\max} + \sqrt{K^2 - \tfrac{1}{2}\,k^1_{\max}\,K}$$

$$\frac{4}{9}\,{k^1}^2_{\max} = K^2 - \tfrac{1}{2}\,k^1_{\max}\,K$$

$$k^1_{\max} = 1\cdot04\ K$$

$$\therefore k^1_{\max} = k_{\max} \times \frac{1\cdot04}{\cdot6} = 1\cdot735\ k_{\max}$$

$$1\cdot2\ T = 1\cdot735 \times \frac{2\frac{1}{2}\ T}{\frac{\pi}{16}\,d^3} \times \frac{\pi}{16}\,d_1{}^3$$

$$\therefore d_1{}^3 = 8^3 \times \frac{1\cdot2}{1\cdot735 \times 2\frac{1}{2}}$$

$$d_1 = 5\cdot22\ \text{in.}$$

Example II.—A cast-iron spur wheel, with 40 teeth of 2 in. pitch, is keyed on a wrought-iron shaft. Assuming that the whole effort is exerted on the corner of one tooth, and that the shaft is subjected to a bending moment of the same magnitude as the twisting moment imparted to it by the spur wheel, determine the diameter of the shaft for equality of strength between it and the spur wheel.

Let n P be the force acting on the corner of a tooth where n is a fraction, and P is the whole force acting on the wheel teeth; the moment of this force about the section where breaking is likely to occur is

$$n\,P\,\frac{h}{\sqrt{2}} = f\,h\,\sqrt{2}\,t^2$$

where t is the thickness of a worn tooth at the pitch line; assume

$$t = \cdot36\,p \qquad n = \tfrac{2}{3} \qquad f = 4000\ \text{lb.}$$

whence $$P = \tfrac{1}{2} \times 4000 \times (\cdot36)^2\,p^2$$

and $$p = 2^4$$

$$\therefore\quad P = 2000 \times \cdot1293 \times 4.$$

The radius of the wheel

$$= \frac{20 \times 2}{\pi}$$

$\therefore$ the twisting moment

$$T = \frac{2000 \times 40 \times {\cdot}1293 \times 4}{\pi} \text{ inch-pounds}$$

$$= 13200.$$

$$T_e = M + \sqrt{M^2 + T^2} = 2{\cdot}414\ T$$

$$= f \frac{\pi}{16} d^3$$

where d = diameter of shaft and f = 5000 lb.

$$d^3 = 32{\cdot}4$$

$$d = 3{\cdot}19.$$

Example III.—Supposing the aftermost length of shafting in a ship to be supported on three equi-distant bearings which are in a straight line, and to be joined to the next by a loose coupling; show that considerable wear in the aftermost bearing may take place without any increase of the bending moment on the shaft, and explain how to determine the extent of the deviation of that bearing from the line of the other two, at which the maximum bending moment will begin to increase.

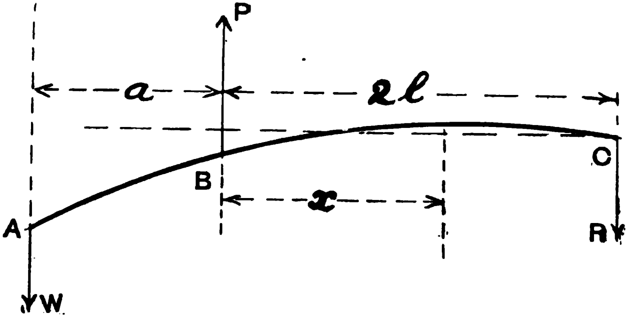


FIG. 89.

Let A B C, fig. 89, be the shaft supported on two bearings at B and C; we shall first find what the upward deflection at the centre of the span B C would be, and thence deduce the downward force Q there that will reduce this deflection to any desired amount.

Let x be the distance measured horizontally from B to any point on B C, and let y be its ordinate measured from a horizontal line through C; then, if M is the bending moment, I the moment of inertia, and E the modulus of the section,

$$\frac{d^2 y}{d x^2} = \frac{M}{E I} = K M = K (W (a + x) - P x)$$

Hence $$c_2 + \frac{d y}{d x} = K\left(W a x + W \frac{x^2}{2} - P \frac{x^2}{2}\right)$$

$$d_2 + c_2 x + y = K\left(W + \frac{a x^2}{2} + \frac{W x^3}{6} - P \frac{x^3}{6}\right).$$

When $x = o$, $d_2 = - y_1$, where y_1 is the distance the bearing B is below the bearing C. When

$$x = 2 l, \quad y = o$$

$$d_2 + 2 c_2 l = K\left(2 W a l^2 + \frac{4 W l^3}{3} - \frac{4 P l^3}{3}\right).$$

When $x = l$ we shall get the deflection at the centre

$$d_2 + c_2 l + y = K\left(\frac{W a l^2}{2} + \frac{W l^3}{6} - P \frac{l^3}{6}\right)$$

$$y + \tfrac{1}{2} d_2 = K\left(W \frac{a l^2}{2} + \frac{W l^3}{6} - \frac{P l^3}{6}\right)$$

$$- c_2 l - \tfrac{1}{2} d_2.$$

$$\therefore y + \tfrac{1}{2} d_2 = - K\left(W \frac{a l^2}{2} + \frac{W l^3}{2} - \frac{P l^3}{2}\right)$$

but $$P = \frac{W (a + 2 l)}{2 l}$$

$$\therefore y + \frac{d_2}{2} = - K \cdot \frac{W a l^2}{4}.$$

Let Q be the force at the centre acting downwards that would cause the deflection at the centre to be zero. Then

$$y = \frac{Q \times (2 l)^3}{48 \text{ E.I.}}$$

$$= \frac{K Q l^3}{6} = - K \cdot \frac{W}{4} a l^2 \tfrac{1}{2} d_2.$$

$$\therefore Q = - \tfrac{3}{2} \frac{W a}{l} - \frac{3 d_2}{K l^3}.$$

After Q has been applied,

$$P = \frac{W(a + 2l)}{2l} - \frac{Q}{2}$$

$$= \frac{W(a + 2l)}{2l} + \tfrac{3}{4}\,\frac{Wa}{l} + \frac{3d_2}{2Kl^3}.$$

$$P = \frac{W}{2l}(2l + \tfrac{5}{2}a) + \tfrac{3}{2}\,\frac{d_2}{Kl^3}.$$

Now, as long as P is greater than W, the greatest bending moment on the shaft will be W a, and when P = W the bending moment will be W a to the middle bearing at which Q acts, but when P is less than W the bending moment will be greater at the centre bearing than at B. Hence the deviation we want is that which occurs when P = W.

$$W = \frac{W(2l + \frac{5}{2}a)}{2l} + \frac{3d_2}{2Kl^3}$$

and

$$d_2 = -y_1$$

$$\therefore y_1 = \tfrac{5}{6}\,\frac{Wal^2}{EI}$$

which is the deflection of B below the line of the other two bearings.

CHAPTER XIII.

FLANGE COUPLINGS.

A FLANGE coupling has to resist torsion; the bolts might give way by shearing, if too small. In the case of a crank shaft which is made in parts that are connected by flange couplings, the bending of the shaft will produce a tensile stress in some of the bolts, as well as a shearing stress. It is not usual, however, to take this into account. Let f_s be the shearing stress in the bolts, and f that produced by torsion in the shaft; then

$$f_s \,.\, n\frac{\pi}{4}\delta^2\frac{D}{2} = f \,.\, \frac{\pi}{16} \,.\, d^3.$$

where n is the number of bolts, and δ their diameter, d the diameter of the shaft, and D that of the bolt circle. If f_s is equal to f,

$$n \,.\, \delta^2 \,.\, D = \tfrac{1}{2}d^3.$$

In marine engine practice, however, there is no fixed rule by which to design the flange coupling, although it would appear reasonable to make both δ and D fixed multiples of d, and the mean value of f_s, calculated from the indicated horse power, varies between 2,300 and 4,700. The following table gives examples of flange couplings for marine-engine shafting.

Where the parts of the crank shaft are to be interchangeable, the number of bolts must be a multiple of 4 for two or four cranks, and of 3 for three cranks.

In concluding the subject of shafting, we must add a few lines on the lengths of journals. In the first place, the laws of friction for lubricated journals under constant or varying pressure are unknown, and it is therefore useless to attempt to theorise on the subject. The reader will obtain plenty of information on this subject from the pages of engineering papers, and such books as D. K. Clark's "Steam Engine"; in short, experience is our only guide.

Crank-shaft diameter.	Screw-shaft diameter.	No. of bolts.	Diameter of bolts.	Diameter of coupling.	Thickness of coupling.	Diameter of bolt circle.
4¾	4¾	6	1¼	10¾	1¼	7¾
6	5¾	6	1½	13	1⅝	9½
7¼	7	6	1⅞	15½	2	11½
8¼	8¼	6	2	17¼	2½	12¾
9¼	9	6	2¾	19	2¾	14⅛
9⅞	..	6	2⅜	18	2⅛	14
11	..	6	2½	21½	3	16½
12¼	..	6	3	23½	4	18
13	12	8	2¼	22	3¾	18
16½	16½	8	3½	32	4¼	25
22½	..	9	4¼	35	6	28¾
23	..	8	4¼	38	5	30⅜

Example I.—Two lengths of shafting are connected by a flanged coupling, in which a sleeve, fig. 90, the internal diameter of which is equal to the diameter of the bolt circle, wraps both flanges, the bolt holes being formed half in the flanges and half in the sleeve. Determine the diameter of each of the ten coupling bolts, which will give to the coupling the same strength as the solid shaft of 14 in. diameter, that of the bolt circle being 17½ in., and the thickness of each flange being 3½ in., the material of the bolts being of the same strength as that of the shaft. (Science and Art Honours Machine Construction Examination, 1897.) We may suppose one shaft to rotate while the other is fixed, and we see that the bolt section sheared through is

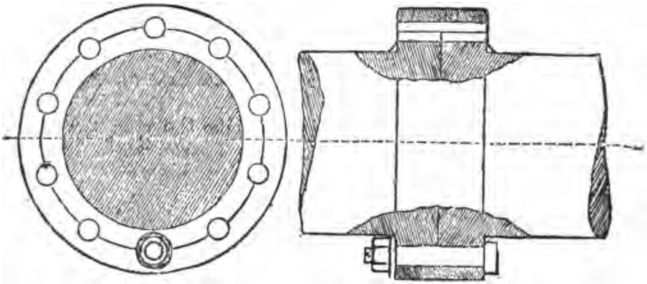


FIG. 90.

$$3\tfrac{1}{2}\,d + \frac{\pi}{8}\,d^2.$$

Hence $$35\,d + \frac{10\,\pi}{8}\,d^2 = \frac{\pi}{16} \times \frac{14^3 \times 2}{17\tfrac{1}{2}}$$

which gives us $$d = 1{\cdot}49\text{ in.}$$

Example II.—Some of the dimensions of a Hooke's coupling are shown in fig. 91. Supposing the shaft to be subject to torsion only, estimate the diameter of the pins which, if they are made of the same material as the shafts, will cause them to be equally liable to yield to the straining actions to which they are subjected.

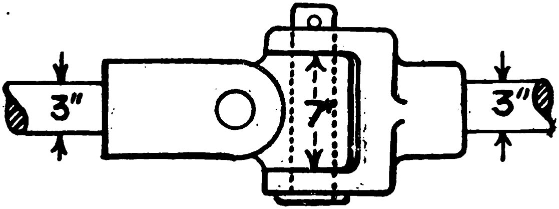


FIG. 91.

The pin will either give way to shear or to bending. Considering the former, let P be the shearing force; then

$$7\,P = f\,\frac{\pi}{16}\,d^3 . = 7\,f\,\frac{\pi}{4}\,d_1{}^2$$

where d_1 is the diameter of the pin. Hence,

$$d_1{}^2 = \frac{4 \times 27}{16 \times 7} = \cdot 965$$

$$d_1 = \cdot 982 \text{ in.}$$

If we suppose the pin gives way by bending, we must assume some point of action for the bending force. Let P_1 be this force, and let its overhang be $\frac{3}{4}$ in. Then

$$8\tfrac{1}{2}\,P_1 = f\,\frac{\pi}{16}\,d^3.$$

$$\tfrac{3}{4}\,P_1 = f\,\frac{\pi}{32}\,d_1{}^3.$$

$$\therefore \frac{8\frac{1}{2}}{\frac{3}{4}} = 2\,\frac{d^3}{d_1{}^3}.$$

$$d_1 = d\,\sqrt[3]{\cdot 1765}$$

$$= \cdot 56\,d.$$

Hence the former diameter is more suitable.

CHAPTER XIV.

RESILIENCE.

THE work done in deforming a bar up to the elastic limit is termed the resilience of the bar. Let A be the cross-section of the bar, and f the stress thus produced; then the work done in producing the deformation is $\frac{1}{2} f \,.\, A \,.\, s$, where s is the extension; but

$$\frac{\text{stress}}{\text{strain}} = E$$

the modulus of elasticity;

$$\therefore \frac{f\,l}{s} = E$$

where l is the length of the bar.

If a force P suddenly acts upon an unstretched bar—*e.g.*, a cap bolt of the bearings of a vertical engine—the work done will be

$$P s_1 = \tfrac{1}{2} f_1 A s_1$$
$$\therefore P = \tfrac{1}{2} f_1 A$$

and produces double the stress that it would cause if gradually applied; for this reason sudden changes of pressure should be avoided as much as possible.

If a bar is subjected to shocks causing extension, its section should be as uniform as other circumstances will allow. If it is compressed by the shocks, the case is different, because its tendency to buckle must be considered.

For this reason the section of the body of a bolt is sometimes reduced so that it may be equal to the section at the bottom of the thread, but it would be unreasonable to make the section of the piston rod equal to the section of the screwed end at the bottom of the thread: firstly, because the former must be given additional strength to resist compression; and secondly, because it is alternately in compression and tension, and would give way with a smaller stress than when subjected to either tension or compression acting alone, while the screwed end is only in tension. The following example will illustrate the advantage in bolts of uniformity of section. Let us compare the resilience per unit volume of two bolts A and B, in which A for one-tenth of its length has a sectional area which is eight-tenths of that of the remainder; and B for nine-tenths of its length has a sectional area which is eight-tenths of that of the remainder; by "length" we mean the part subject to the full load—*i.e.*, from the point where it joins the head.

Let f be the safe stress to which each can be subjected; let a be the area of the larger part of the bolt; then the greatest total stress in each case is $\cdot 8 f a$, giving a stress f per square inch on the smaller section.

Let $10 l$ be the length of each bolt and s the extension of A; then

$$s = \frac{fl}{E} + \frac{\cdot 8 f \times 9 l}{E} = 8{\cdot}2\frac{fl}{E}$$

Let F be the force gradually applied to A or B that would produce the stress f in the smaller part; the resilience of A is

$$\tfrac{1}{2}\,F\,s = \tfrac{1}{2}\,F \times 8{\cdot}2\frac{f\,l}{E}.$$

$$= 4{\cdot}1\,\frac{F\,f\,l}{E}.$$

Similarly, the resilience of B is

$$4{\cdot}9\,\frac{F\,f\,l}{E}.$$

The volumes of A and B are $9{\cdot}8\,a\,l$ and $8{\cdot}2\,a\,l$ respectively, and therefore the resilience per unit volume of A is—

$$\frac{\text{resilience of A}}{\text{volume of A}} = \frac{4{\cdot}1\,.\,F\,.\,f\,.\,l}{9{\cdot}8\,.\,a\,l\,.\,E}$$

and

$$\frac{\text{resilience of B}}{\text{volume of B}} = \frac{4{\cdot}9\,F\,f\,l}{8{\cdot}2\,a\,l\,E}$$

$$\therefore \frac{\text{resilience per unit volume of A}}{\text{resilience per unit volume of B}} = \left(\frac{8{\cdot}2}{9{\cdot}8}\right)^2 = {\cdot}7$$

showing that, weight for weight, the first bolt has only ·7 the power of resistance to shock of the second.

The following question was given in an examination in machine construction of the Science and Art Department; although not of a very practical nature, it is interesting: "Suppose a machine or other structure to be strained, within the limit of elasticity, by a load which is suddenly reversed in direction, but unaltered in magnitude; and suppose the reversal to be repeated each time at the instant when the structure has become most strained by the previous application of the load. Show that the stress produced after n applications of a load P, to an originally unstrained structure, is equal to that due to the steady application of a load at $2\,n$ P."

The difficulty in answering this question is that it requires an inductive method of treatment, the knowledge of which, although useful to a mathematician, is not likely to be of service to an engineer. We have already shown that the proposition is true when n equals unity. Let us assume it to be true when n has some value k; we shall then prove its truth when n is $k + 1$, and thus establish the proposition for all values of n.

We shall only consider the case when the structure is a bar upon which a longitudinal force acts. By similar reasoning the reader can prove the truth of the proposition for a beam, shaft, &c., and thus establish it for any structure. We assume, then, that

$$f_k\, a = 2\, k\, P.$$

where f_k is the stress produced by the kth application of P. Then when P is again reversed

$$(P + \tfrac{1}{2} f_k\, a)\, .\, s_k = \tfrac{1}{2} f_{k+1}\, .\, a\, .\, s_{k+1} - P\, .\, s_{k+1}\, .$$

where a is the sectional area of the bar, s_k is the kth elongation or compression, and s_{k+1} that following. The above equation is obtained thus: Suppose the kth application causes elongation, the work done by P after reversal is P $(s_k + s_{k+1})$, and by the resilience of the bar is $\tfrac{1}{2} f_k\, a\, s_k$; the work done on the bar is $\tfrac{1}{2} f_{k+1}\, a\, s_{k+1}$, and the sum of the first two quantities equals the third; but

$$\frac{s_k}{f_k} = \frac{s_{k+1}}{f_{k+1}}$$

$$\therefore (k + 1)\, P\, .\, 2kP + P\, .\, f_{k+1}\, .\, a = \tfrac{1}{2} f^2_{k+1}\, a^2$$

a quadratic from which we obtain

$$f_{k+1}\, .\, a = 2\,(k + 1)\, P\, .$$

which proves the proposition.

THE RESILIENCE OF A SHAFT.

Let r be the radius C D of a shaft, fig. 92, and let it be twisted, being fixed at the end E so that the line B C is

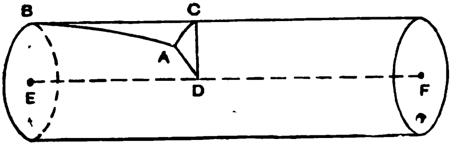


FIG. 92.

displaced to B A, forming a spiral. Then A D C is θ, the angle of torsion for a length l, where B C = l. Then if f is the stress caused by torsion, and G the modulus of transverse elasticity,

$$G\, r\, \theta = f\, .\, l\, .$$

$$\theta = \frac{2 f l}{G d}$$

d being the shaft diameter; but

$$f = \frac{T}{\frac{\pi}{16} d^3}$$

$$\therefore \theta = \frac{2\,T\,l.}{G\,\frac{\pi}{16}\,d^4} = \frac{32\,T.\,l}{\pi\,G\,d^4}.$$

The following are values of G: for wrought iron 10,500,000, for cast steel untempered 12,000,000, and if tempered 14,000,000.

If we have, as is sometimes the case, to consider the stiffness rather than the strength of a shaft, we must use the above equation. According to Professor Unwin,

$$d = \beta \sqrt[4]{\frac{H.\,P.}{N.}}$$

The following table gives values of β:—

	Steel.	Wrot. iron.	Cast iron.
Stress changing little during work, and not reversing..	2·877	3·294	4·469
Part of stress reversing at each revolution............	3·294	3·770	5·116
Stress changing constantly between equal and opposite values	4·149	4·749	6·446

The resilience per cubic inch of a cylindrical bar, subjected to tension only.

Let A be the section and P the force that will stretch the bar to the elastic limit at which the stress is f_1; then the total work done is

$$\tfrac{1}{2} f_1 \,.\, A.\, s. = \text{resilience}$$

where s is the extension in inches,

but
$$\frac{f_1\,l}{s} = E$$

where l is the natural length of the bar, and E is the modulus of elasticity in pounds;

$$\therefore \text{resilience} = \tfrac{1}{2} f_1\, A \frac{f_1\,l}{E}.$$

$$= \tfrac{1}{2} f_1^{\,2} \frac{A\,l}{E}.$$

so that the resilience per unit volume is

$$\frac{f_1^2}{2E}.$$

The resilience of a rectangular bar supported at both ends and loaded in the middle, and so designed that the maximum stress f_1 is produced at every section and the curvature is constant.

Let $l = 2a$ be the span, W the central load, b the breadth, h the depth, and r the radius of curvature at any point,

$$M = \frac{EI}{r}.$$

where I is the moment of inertia of the section.

$$\frac{f_1 \, b h^2}{6} = \frac{E \, b h^3}{12 r}.$$

$$f_1 = \frac{E h}{2 r} \quad \ldots \ldots \quad (1)$$

$$\frac{1}{r} = \frac{d^2 y}{d x^2}$$

where x is the distance from the centre and y is the height of the centre line from a tangent to the centre of the beam, y being very small.

$$\therefore r = \frac{E h}{2 f_1}$$

$$\frac{d^2 y}{d x^2} = \frac{2 f_1}{E h}.$$

It is clear that h is constant.

$$\frac{d y}{d x} = \frac{2 f_1 x}{E h} \text{ and } y = \frac{f_1 x^2}{E h}.$$

$\therefore \delta$, the deflection at the centre,

$$= \frac{f_1 a^2}{E h}.$$

Again,

$$\frac{W}{2}\left(a - x\right) = f_1 \frac{b h^2}{6}.$$

$$\therefore b h = \frac{3 W (a - x)}{f_1 h}.$$

Hence the total volume of the beam is

$$2\int_o^a \frac{3\,W\,(a - x)}{f_1\,h}\,d\,x.$$

$$= \left[\frac{6\,W\left(a\,x - \frac{x^2}{2}\right)}{f_1\,h.}\right]_o^a$$

$$= \frac{3\,W\,a^2}{f_1\,h.}$$

Hence the resilience per unit volume is

$$\frac{\frac{1}{2}\,W.\delta.}{3\,\frac{W\,a^2}{f_1\,h.}}$$

$$= \frac{\delta\,f_1\,h}{6\,a^2} = \frac{f_1{}^2\,a^2\,h.}{6\,a^2\,E\,h.}$$

$$= \frac{f_1{}^2}{6\,E}.$$

The same formula applies to all rectangular beams of uniform strength, to coach springs if properly designed, and to spiral springs subjected to a moment whose axis coincides with the axis of the spiral.

The resilience per unit volume of a cantilever loaded at the end, and of uniform section $b\,h$.

Let x be the distance of a point from the end, and y the small height of the point from the lowest point of the beam. Then

$$M = E\,I.\frac{d^2\,y}{d\,x^2}. = W\,x.$$

$$E\,I\,\frac{d\,y}{d\,x} = W\,\frac{x^2}{2} + C.$$

$$\text{and C evidently} = -\,\frac{W\,l^2}{2.}$$

$$\therefore\ E\,I\,\frac{d\,y}{d\,x} = -\,\frac{W}{2}\,(l^2 - x^2)$$

$$E\,I\,y = -\,\frac{W}{2}\left(l^2\,x - \frac{x^3}{3}\right) + k.$$

but when $y = o$, $x = o$; hence $k = o$.

$$E\,I.\delta. = -\,\frac{W}{2}\,\tfrac{2}{3}\,l^3 = -\,\frac{W.l.^3}{3}$$

The resilience per unit volume is therefore

$$\tfrac{1}{2}\,\frac{W\,\delta.}{b.h.l.}$$

$$= \frac{W^2\,l^3}{6\,E\,I.b.h.l.}$$

$$= \frac{f_1{}^2}{18\,E}.$$

This also applies to a beam supported at the ends, and loaded in the middle if of uniform section.

The resilience of a solid cylindrical shaft.

Using the notation of page 121, we have

$$\text{resilience} = \tfrac{1}{2}\,T.\theta.$$

$$= \frac{\pi\,f^2\,d^2\,l}{16\,G}$$

$$\therefore\ \text{resilience per unit volume} = \frac{f^2}{4\,G}.$$

This also applies to a cylindrical spiral spring of wire of circular section.

The resilience of a solid square shaft.

In this case

$$T = \frac{f\,s^3}{4{\cdot}79}.$$

where s = side of square, so that the resilience per unit volume

$$= \frac{\tfrac{1}{2}\,T\,\theta}{s^2\,l} = \frac{f.s.\theta.}{9{\cdot}58\,l}$$

Again, $$\theta = \frac{T.l.}{{\cdot}141\,G.s.4}.$$

Hence the resilience per unit volume is

$$\frac{f^2}{6{\cdot}475\,G.}$$

This also applies to a cylindrical spiral spring of square section.

CHAPTER XV.

EXPANSION VALVE GEARS.

THERE are many forms of expansion valve gear. We shall first consider those in which a valve called the "expansion valve" works on the back of another called the "distribution valve." A theoretical sketch is shown in fig. 93. Here A, B are the two plates of the expansion valve, and C is the distribution valve—so called because it distributes the steam to the steam passages and controls the exhaust. There are two eccentrics, one for each valve, and there is, in all cases, some mechanism by means of which the point of cut-off may be varied.

1. The plates A, B may be put farther apart, the cut-off being effected by their outside edges.

2. The travel of the expansion valve may be altered, and the cut-off may be made by the outside or inside edges of A, B.

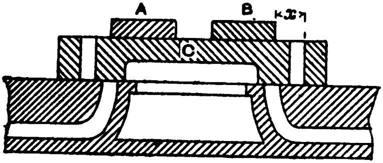


FIG. 93.

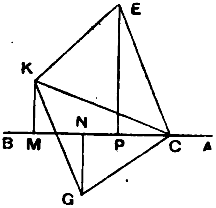


FIG. 94.

3. The angle of advance of the expansion eccentric may be altered without altering its throw.

By the word "throw," we mean the distance between the centres of the shaft and of the eccentric.

In fig. 94, if A B is the line of stroke, the direction of rotation being opposite to that of the hands of a clock, and if C, E, K are the centres of shaft, distribution eccentric, and expansion eccentric, and if K M, E P are perpendiculars on A B, then the centre of the expansion eccentric is at a distance P M to the left of the centre of the distribution eccentric; so that if K G, C G are drawn parallel to C E and K E respectively, and G N is drawn perpendicular to A B, C N will always represent the distance between the centres of the two valves. The relative motion of A, B to C is the same as would be obtained if C were fixed and A, B connected to an eccentric having an angle of advance greater than that of the distribution eccentric by the angle G C E. Suppose G C A, an angle greater than 180 deg., is its angle of advance plus 90 deg., and in fig. 95 take B C G^1, B C E, B C K equal to A C G, A C E, A C K, fig. 94; then when the crank is in any position such as C D, the expansion valve will be C F to the *left* of its central position relative to C, and if the crank is at C H, then C L will be the distance between their centres, but the expansion-valve centre will be to the *right* of that of the distribution valve. Given the angles of advance of the two eccentrics, the construction of a valve diagram is easily made; make B C E, B C K equal to the angles of advance plus 90 deg.; draw E G, C G parallel and equal to C K, E K; take C G^1 equal to C G, and describe circles upon them as diameters.

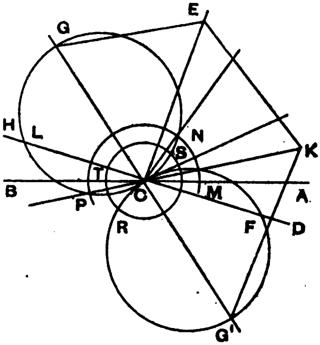


FIG. 95.

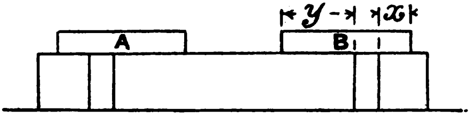


FIG. 96.

This diagram enables us to solve problems in expansion-valve gears without having to consider the absolute motion of each valve. If C M is made equal to x, fig. 93, and a circle M N P be drawn, it is clear that the plate B will close the port of the distribution valve when the crank is at C N, and re-open it when the crank reaches C P. If, when the centres of the valves coincide, the outside edges of A and B overlap the outside edges of the steam passages, fig. 96, and if C T be taken equal to x, and a circle T R S be described, then steam will be cut off from the crank position C S to C R, and the passage will be opened from R to S, S being on the circle whose diameter is CG^1.

If it is intended to cut off steam by the inner edges of the expansion plates, take C M, fig. 97, equal to y in fig. 96, and draw a circle N M P with C as centre; then the passage in the distribution valve will open when the crank is at C N, because the expansion valve is now a distance y from its relatively central position, and the passage will again close when the crank reaches C P, C G being the diameter of the resultant circle whose chords through C give the relative motions of the two valves.

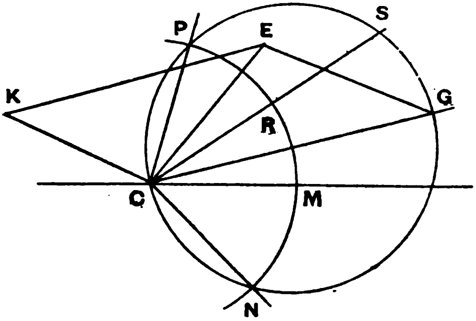


FIG 97.

The port opening for any position C L of the crank is shown in fig. 98 by the part of the radius C L outside the circle C G, and inside the circle M N. If the crank line cuts the circle CG^1, then the port opening is C R + C S. Of course, the port opening can never be greater than the passage in the distribution valve, so that the above show the distances between the outer edges of the valve plate and port. In fig. 97, R S is the port opening for the crank position C S.

The Meyer Expansion Valve.—In this valve the cut-off is altered by separating the plates by means of a right and left handed screw passing through nuts fixed to the two plates. In fig. 98 it is clear that by decreasing C M the radius C N may be brought nearer to C A; when C M is zero, cut-off takes place when the crank is at C T. If the lap be made positive the cut-off is still earlier, as explained above, and illustrated by fig. 95. The latest cut-off occurs when C G is the negative lap—*i.e.*, x in fig. 93.

We shall next consider the rate of reduction of passage area at cut-off. Let ω be the angular velocity of the shaft, and b the breadth of the passage of the distribution valve; then when cut-off occurs at N the valve has moved a distance Q L when the shaft has turned through the angle N C L, fig. 98. The rate of reduction of passage area at cut-off is—

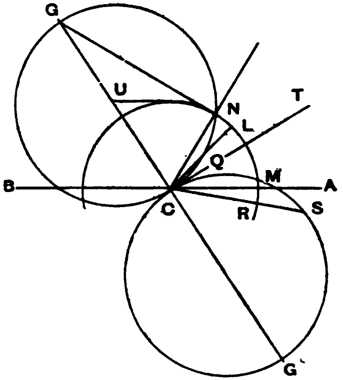


FIG. 98.

$$V = \frac{\mathrm{Q\,L}\,.\,b}{\text{time required to turn through the angle N C L}}$$

when the distance N L is indefinitely diminished.

$$V = \frac{\mathrm{Q\,L}\,.\,b\,.\,\omega}{\frac{\mathrm{N\,L}}{\mathrm{C\,L}}} = x\,b\,\omega \tan \mathrm{C\,N\,U} = b\,.\,\mathrm{N\,G}\,.\,\omega,$$

because the angle C N U is equal to the angle Q N L when N L is indefinitely diminished. This is one method of measuring the velocity of cut-off, but absence of wire-drawing does not depend on the magnitude of *b* N G. A slow cut-off when the piston is moving slowly may be better than a more rapid cut-off when the piston is moving faster. A better method of measuring the comparative rapidity of the cut-off is to take A B, fig. 99, to a small scale to represent the stroke of the piston, and to draw vertical lines to represent the area of port opening at each point of the stroke. Thus in fig. 99 A D is the lead, and the cylinder port is opened from D to E, when the openings of cylinder port and of distribution-valve passage are equal, and the latter is being reduced until cut-off takes place at C.

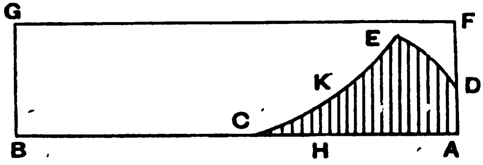


FIG. 99.

It is clear that the larger the angle K C H, the better the cut-off, for K H is the reduction of port area at the same time that the piston moves through C H; so that tan K C H is the ratio of velocity of cut-off to piston velocity at cut-off, and this measures the value of the cut-off or absence of wire-drawing.

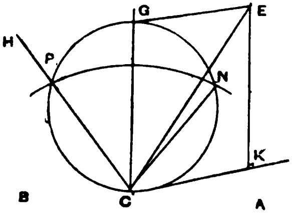


FIG. 100.

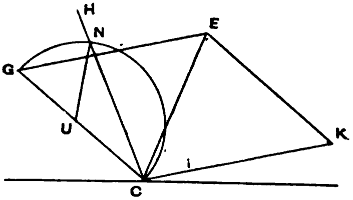


FIG. 101.

It is important that the throw and angle of advance of the expansion eccentric should be so chosen that the expansion valve does not re-open the passages before the distribution valve has cut off the steam. A faulty design is shown in fig. 100, where C H is the position of crank when the latter cuts off steam, and C G is the resultant circle. Then, if the negative lap is made greater than C N or C P, the cut-off will be later than C N, but the expansion valve will re-admit steam before C P.

If, as in fig. 101, C G lies on the other side of C H, or if it coincides with it, this fault will not exist. In Holmes' "Steam Engine" we find it stated that C G should coincide with C H, the position of the crank at cut-off, by the distribution valve; but we do not always find it so in practice, because this will make cut-off take place very slowly when the expansion and distribution valves cut off simultaneously, because N G will then be zero, fig. 101. If the normal cut-off is earlier in the stroke (say at about one-third), and the late cut-off is only used for starting the engine, this is of no consequence, and will not affect the efficiency of the engine.

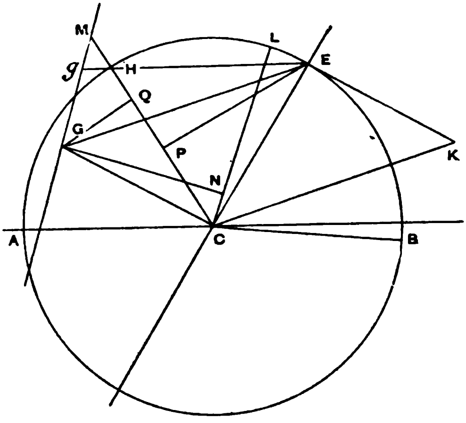


FIG. 102.

If in fig. 102 the normal cut-off takes place when the crank is at C L, we should make the velocity of cut-off equal to that of the distribution valve. Draw M G at the same distance from C L that E is from C H, the position of crank when the distribution valve cuts off. Then any point G on M G will make N G, which measures the velocity of cut-off of the expansion valve, equal to E P, which measures that of the distribution valve. The least velocity of cut-off is measured by G Q, the perpendicular on C H, this being the velocity when C Q is the negative lap of the expansion valve, and cut-off takes place at C H. The greater we make G Q, the larger C K becomes; in the figure we have made G Q equal to half G N.

If the engine runs as often in one direction as the other, the line C K should coincide with A B, and E *g* parallel to A B will give the throw of the expansion eccentric. Having settled the position of the expansion eccentric, we must next find the least negative lap of the expansion valve, or its greatest overlap, to give the earliest cut-off that is required, and which is usually at the beginning of the stroke. Suppose (fig. 95) that the earliest cut-off is to take place when the crank is at C N, then C N is the least negative

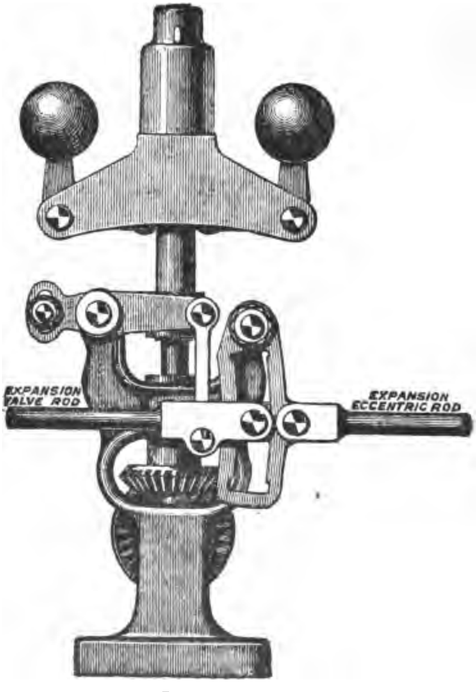


FIG. 103.

lap; or, if the crank is at C S, then C S is the greatest positive lap. The plates of the expansion valve must not be too short, or the passages may be re-opened by the inner edge of the plates reaching them. To find their correct length, we remember that the relative half-travel of the expansion valve is C G, so that the right-hand plate, fig. 96, will move C G to the right, so that the length of plate must be x + width of passage + C G + about $\frac{1}{8}$ in., to prevent the passage re-opening; if x is negative, then we may subtract it. The inner ends of the plates should touch one another when the negative lap has the greatest value intended.

Hartnell and other Expansion Gears.—The second method of altering the cut-off is by varying the travel of the valve, the lap being unaltered. The cut-off may be made by the outside or inside edges of the plates. An example of the former is the Hartnell automatic expansion gear, fig 103, in which the eccentric rod end is fixed to a slot, pivoted at one end, in which a block can be raised or lowered by the governor. To the block is connected the expansion valve rod, so that the lower the block in the slot, the longer the travel of the valve and the later the cut-off. We shall now consider the theory of this gear.

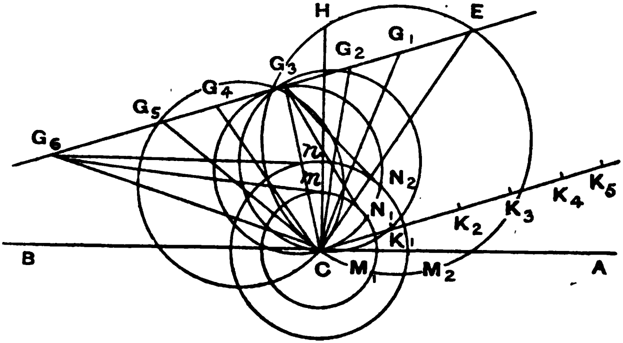


FIG. 104.

In fig. 104, let B C E be drawn equal to the angle of advance of the distribution eccentric, plus 90 deg., and B C K_5 be a similar quantity for the expansion eccentric. We may begin by assuming that the two eccentrics have equal throws. Take E G_1, E G_2, &c., on E G_5, which is parallel to C K_5, equal to C K_1, C K_2, &c., respectively; then C G_1, C G_2, &c., are the resultant circles, when C K_1, C K_2, &c., are the half travels of the expansion valve. With C M_1 as negative lap of the plates of the expansion valve, it is clear that the cut-off is later the longer the travel of that valve. This will be seen at once if circles are described with C G_1, &c., as diameters.

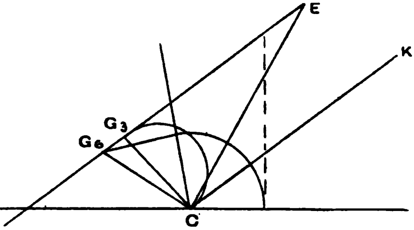


Fig. 105

We shall first consider the effect of varying the three important quantities in the design, viz., the negative lap of the expansion valve, which is generally made double-ported, and the angles of advance of the eccentrics. C G_3 is drawn perpendicular to E G_5, and, with C M_1 as lap, the velocity of cut-off is proportional to G_3 N_1, and this is its least velocity, and the latest cut-off is when the crank is at C G_2, supposing that C G_6 is the greatest relative half travel allowed and G_6 *m* is perpendicular to C G_2, C *m* being equal to C M_1. Suppose the lap increased to C M_2, then the least velocity of cut-off is reduced and is proportional to G_3 N_2; but, on the other hand, the latest cut-off possible is at C H. In fig. 105 is shown the effect of

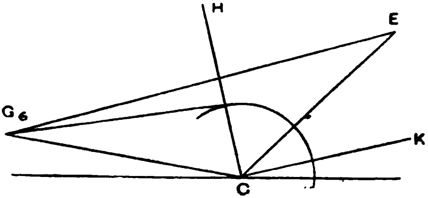


Fig. 106.

reducing the angle K C E. This reduces C G_3, and therefore the least velocity of cut-off; but if the latest cut-off is fixed, this reduction lessens C G_6.

Suppose E C K and the lap fixed, then an increase of the angle of advance of the distribution eccentric increases C G E, without any corresponding advantage, so that the angle of advance of C E should be small. In order to obtain this, we may make cut-off by the distribution valve take place at about ·8 of the stroke. In fig. 107 this has been done, and the expansion valve so designed that cut-off can take place anywhere from the commencement to six-tenths of the stroke. The angle of advance of C K is 60 deg., and

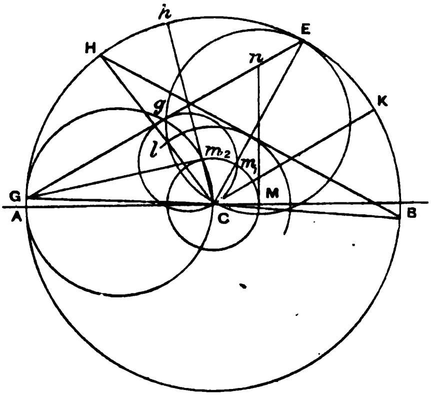


FIG. 107.

E G is drawn parallel to C K; from G on the circle G m_2 is drawn perpendicular to C h, giving the negative lap. Cg is perpendicular to E G, and when the half travel of the expansion valve is Eg the cut-off takes place with least velocity, the crank being at C m_1; if Cl is the outside lap, l being on C H, then the velocity of cut-off of the distribution valve is proportional to El, and of the expansion valve, which is double-ported, to twice $g m_1$. It will be found on measuring the figure that

$$2 g m_1 = \mathrm{E} l,$$

so that these velocities are equal.

It must be possible to reduce the half travel of valve to En, Mn being perpendicular to A B, if cut-off is to take place at the commencement of the stroke; if its least value is greater, the earliest cut-off will be only a trifle later. The greatest relative travel of the two valves has been made equal to that of the distribution valve, and its greatest actual travel is 1·7 that of the distribution valve, being 2 E G. In fig. 108 *a m* represents the stroke of the piston, and *a n* the full port opening of the distribution valve; any ordinate to the curve *b c d* is its port opening for a point of stroke represented by the abscissa, and the dotted lines show by their ordinates the port openings obtained when the expansion valve cuts off at *l*, *h*, and *f*, the valve being double-ported. It will be seen that the ratio of velocity of cut-off to velocity of piston is greater than that which would be obtained by cut-off with the distribution valve.

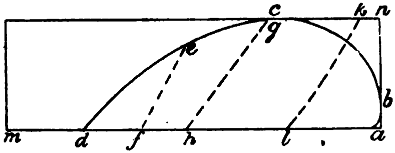


FIG. 108.

We have already stated that cut-off may be effected by the inside edges of the plates of the expansion valve. These have positive lap, and the cut-off is altered by increasing or reducing the travel of the expansion valve. If C G, fig. 109, is the resultant circle for the right-hand passage, admission takes place when the crank is at H and cut-off at K, C H being the lap, because the chords of this circle drawn through C give the relative motion of the expansion valve to the right of its central position; and when it is

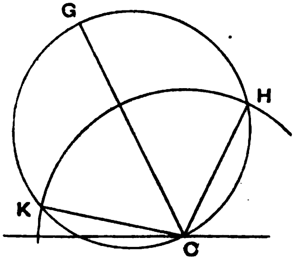


FIG. 109.

C H, or the lap to the right, and moving to the right, admission commences, and when moving to the left, cut-off is just taking place.

In fig. 110 let A E B be the direction of rotation, and B C D the angle of advance of the distribution eccentric, plus 90 deg. Let C D be its throw, and C E the position of the crank at earliest cut-off. Take C H, the lap of the expansion valve on C E, and draw H O a perpendicular to C E. It is usual to take C H equal to the lap of the distribution valve. With centre D, and radius D O, the throw of the expansion eccentric, draw an arc, cutting H O in the point O, and join C O. Complete the parallelogram O D T C. Then T C B is the angle between the centre line of the expansion eccentric and the crank. D O is generally taken equal to C D. When the travel of the expansion valve is reduced to zero, the cut-off by the expansion valve is the same as by the distribution valve, and as the travel increases to twice C T, then C O is the diameter of the resultant circle, and C H is the position of the crank at cut-off. If C U is the half travel of the expansion valve, and C V is drawn

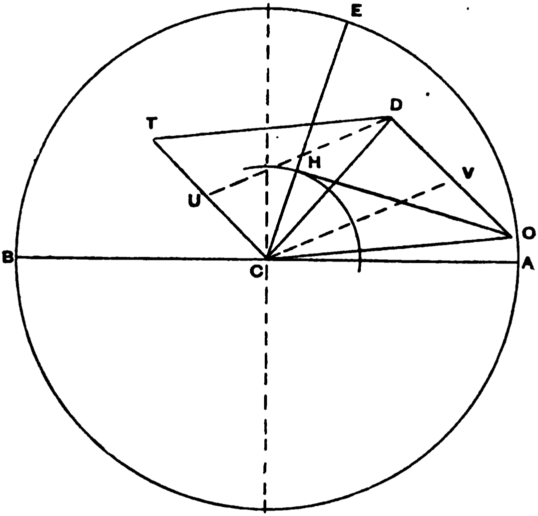


FIG. 110.

parallel to D U, then C V is the diameter of the resultant circle. When C U D is a right angle, the expansion valve has its least velocity of cut-off, and the opening of the steam passages in the distribution valve is least.

The third method we mentioned above was that of altering the angle of advance of the expansion eccentric without increasing or decreasing its throw.

In fig. 111 C E represents the distribution eccentric, C K, C K_1 two positions of the expansion eccentric. C G and C G_1 are parallel, and equal to K E and K_1 E, and are the diameters of two resultant circles. When the expansion eccentric is in the position represented by C K, cut-off takes place when the crank is at C G, the negative lap of the expansion plates being C G. When the angle of advance is increased by K C K_1, cut-off takes place when the crank is at C H. In order to rotate the eccentric on the shaft a flywheel governor is generally used, such as that shown in fig. 112. Here C is the eccentric sheave, A, A are weights

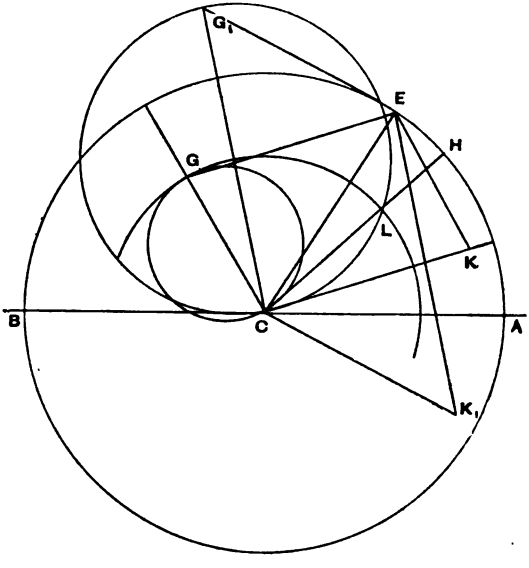


FIG. 111.

fixed at *b*, *b*, and connected to the sheave by the links *e*, *g*, so that when they fly out C is rotated. The springs F, F oppose the weights, and in order to compensate for the varying leverage of A, A, the springs P, P are added. These are also intended to overcome the friction of the valves and valve gear.

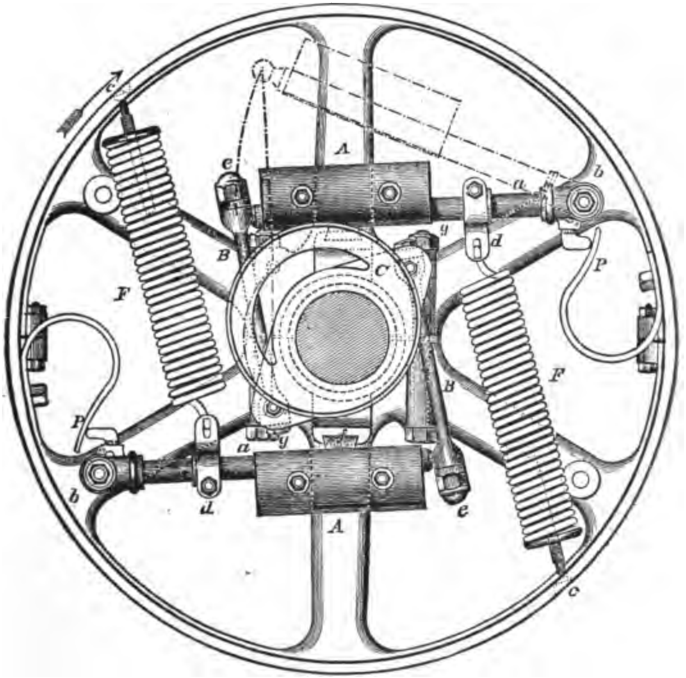


Fig. 112.—Flywheel Governor.

We shall next consider two valve gears in which only one eccentric and valve are used. In the first of these the eccentric is formed of a sheave C, fig. 113, and a ring D, the centre of the latter determining the motion of the valve. The two levers 1, 1 are connected by the rods 2, 2 to C, and by 3 to D, so that when the levers fly out, fig. 114, D is moved in the direction of the hands of a clock, and C in

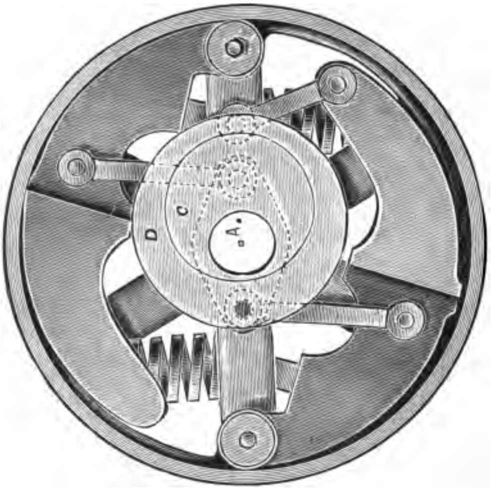


FIG. 114.—Armington-Sims Governor.

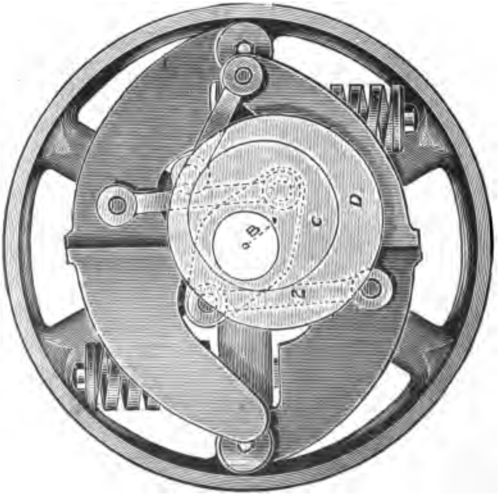


FIG. 113.—Armington-Sims Governor.

the opposite. Thus the throw and angle of advance of the ring D can be altered. This is the arrangement in the Armington-Sims high-speed automatic cut-off engine.

In order to keep the lead constant, fig. 115, the point *e*, the centre of the ring D, fig. 113, must be moved along the line E D, where C D is the lap, plus lead of the valve, and E is the position of the centre of the ring when C, F, and E are in a straight line and the valve has maximum travel. In the figure, C B is the position of the crank, as in this engine a piston valve, admitting steam from between the pistons and exhausting it at the outside of them, is used.

The second valve gear is that used in the Westinghouse high-speed engine. The eccentric and flywheel governor are shown in figs. 116 and 117, the former showing the sheave in the position for maximum travel of the valve, and the latter the minimum. The two levers B, B are pivoted at *b*, *b*, so that when they fly out they pull the sheave C across the shaft S about its pivot *d*. The springs D oppose the action of the weights, so that the parts are as in fig. 116 until the engine is within a few revolutions of its full speed. The centrifugal force of the weights then overbalances the force of the springs, and the throw of the eccentric is reduced, while the angle of advance is increased.

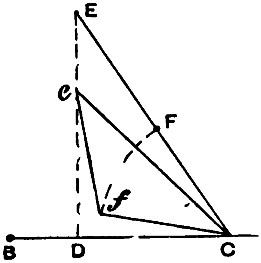


FIG. 115.

In fig. 118 A B is the greatest travel of the valve, and G C E the angle of advance of the eccentric when it gives the valve this travel; E D, an arc of a circle, is drawn with a radius equal to the distance from the centre of *d* to the centre of the sheave, fig. 116. The lead thus increases with the number of expansions, for if C H is the radius of the lap circle, C D − C H is the lead when the centre of the sheave is at D, while C L − C H is the lead when the centre

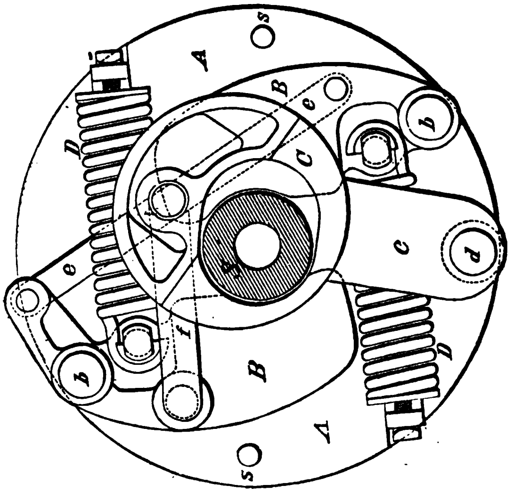


FIG. 116.—Westinghouse Governor.

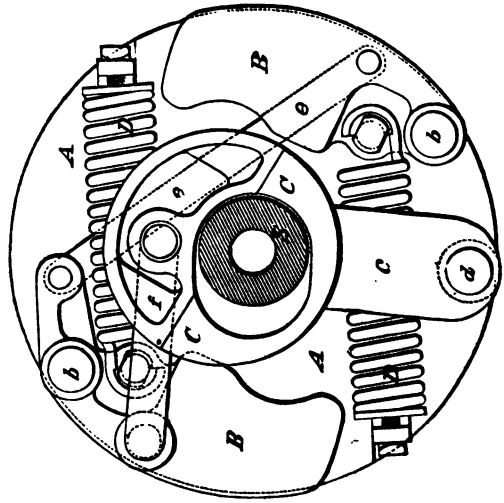


FIG. 117.—Westinghouse Governor.

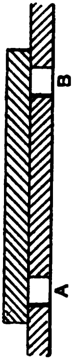


Fig. 119.

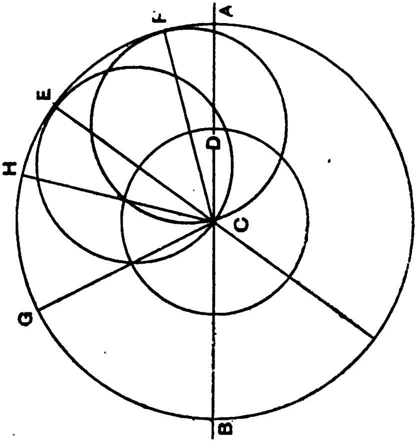


Fig. 120.

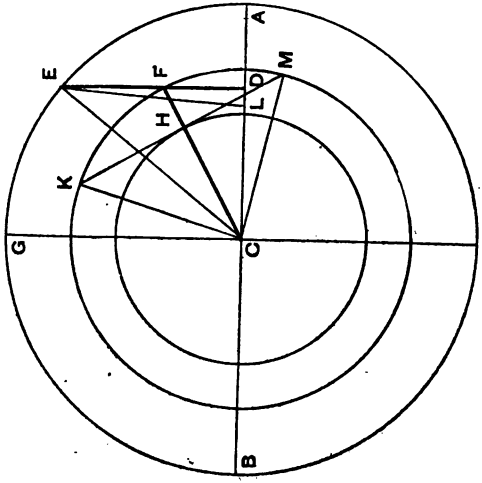


Fig. 118.

is at E. To find the points of admission and cut-off when the throw is C F, make K H M a tangent to the lap circle at H, cutting a circle through F in K and M; then C M, C K are the positions of crank at admission and cut-off.

In another form of expansion valve gear there are two valves; one of these is an ordinary slide valve, and the other moves not on the back of this valve, but over fixed passages A, B, fig. 119. The cut-off is altered by changing the angle of advance of the expansion eccentric. When this is the same as that of the distribution eccentric, the cut-off and admission by both valves will be simultaneous, if both valves have the same travel and outside lap, but when the angle of advance is increased, both admission and cut-off by the expansion valve will be sooner.

In fig. 120 B C E is the angle between crank and distribution eccentric, and the latest cut-off is when the crank is at C G; when the angle of advance of the expansion eccentric is B C F minus 90 deg., cut-off is at C H.

THE BUCKEYE VALVE GEAR.

In this valve gear the expansion valve works on the back of the distribution valve, but by an arrangement of links its relative motion is so arranged that it is always equal to twice the throw of the expansion eccentric. Fig. 121 explains this. Here *g* is the distribution eccentric which drives the distribution valve direct by means of the upper valve spindle, within which is shown the expansion valve spindle which is actuated by the lever *f d c*, *d* being a pivot on the lever *e b*, whose fulcrum is *e*. Neglecting obliquity, the point *d* has a horizontal displacement in the same direction as the distribution valve and of half its amount, and as $fd = dc$, *f* is moved forward a distance equal to twice the movement of *d* from its middle position, added to the motion of *c* from its middle position, this latter, however, being reversed in direction. Hence the relative motion of the expansion valve to the distribution valve is equal and opposite to the horizontal projection of *h*'s motion from its central position. In order to vary the cut-off, the angle of advance of *h* is altered, and the expansion valve diagram is shown on fig. 121; the nearer C E comes to C A, the earlier the cut-off C H. Forms of this valve are shown in figs. 122 and 124, and a valve diagram, with ordinates, valve openings, and abscissæ parts of the stroke, in fig. 123.

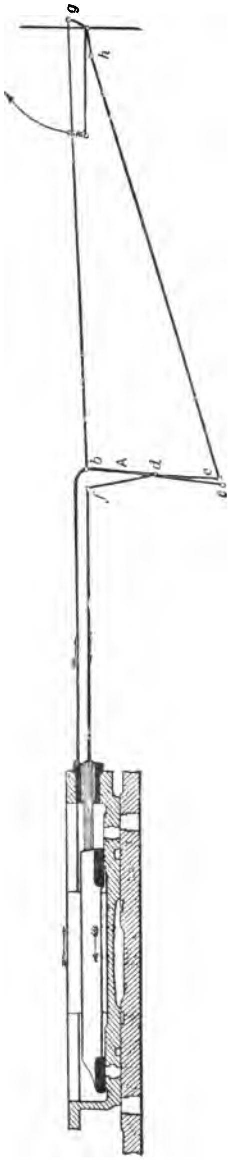


FIG. 121.

STEVENSON'S LINK MOTION.

An approximate solution for this is shown in figs. 125 and 126. If the link be open-armed—that is to say, if the crank pin is at its furthest from the cylinder when the arms do not cross—the construction is shown in fig. 125, but if the arms are crossed, in fig. 126. Draw C L, C L′, L L′ equal to the lengths of rods and links respectively, and C E, C E′ to represent the positions and throws of the eccentrics. Draw E D B perpendicular to C L, and draw an arc E B E′. Then, if E B be divided in any ratio at *e′ e″ e‴* and O L in the same ratios, the lines C *e′* C *e″* C *e‴* give the equivalent eccentrics when the block is in the link at the second three points.

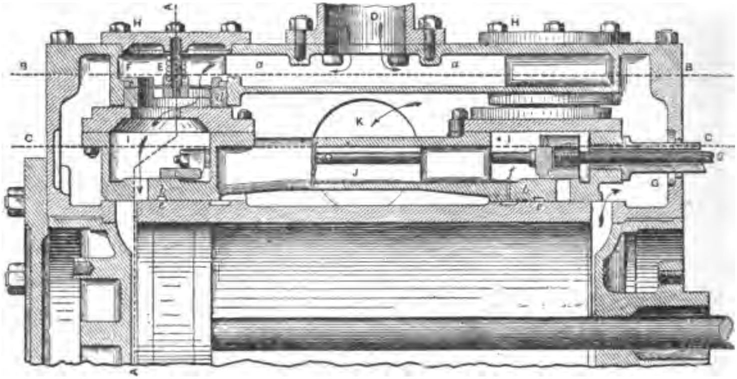


FIG. 122.

By equivalent eccentric we mean that which would give to the valve a motion very similar to that which it actually has.

It will be noticed that the lead is greatest in mid-gear if the arms are open, and least if they are crossed. When a link motion has been designed, an exact method of drawing the motion of the valve for any position of the reversing lever is shown in fig 127, and is due to Messrs. Coste and Maniquet.

Let *o* be the shaft and *o f a d b* the link work, while *h g* is the suspension link. Make a template *r s t*, whose arc *r s* is struck with a radius equal to that of the eccentric rod. Mark off O O equal to this radius, and draw *o f′*, *o b′* parallels to *o f*, *o b* ; then, if the template be laid with *s t* on *f f'*, *a* will lie on the curve *r s*. Also, if *t s* be applied to *b b'*, *d* will lie on *r s*.

Now, if a number—say 16 positions of each eccentric—be marked, and arcs drawn through all such points as *f'*, *b'* in the manner described, and a tracing of the centre line of the link *a g d* be applied to the figure, so that *a* and *d* lie on the arcs, and *g* on the circle whose centre is *h*, then the valve block is at *p*, and the motion of this is the motion of the valve. It will therefore be unnecessary to draw O *f*, O *b*, *f a*, *f d*, and thus much space is saved; but the horizontal distance of *h* from the right-hand O is less than its horizontal distance from the left-hand O by an amount equal to the length of the eccentric rod.

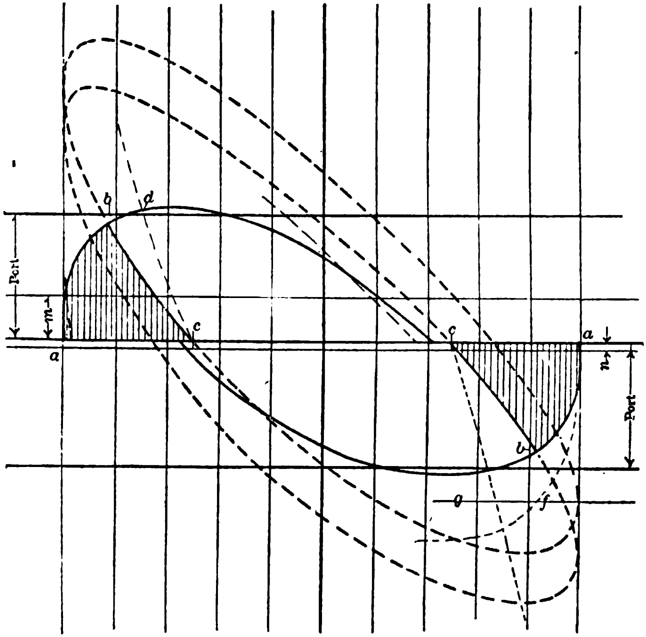


FIG. 123.

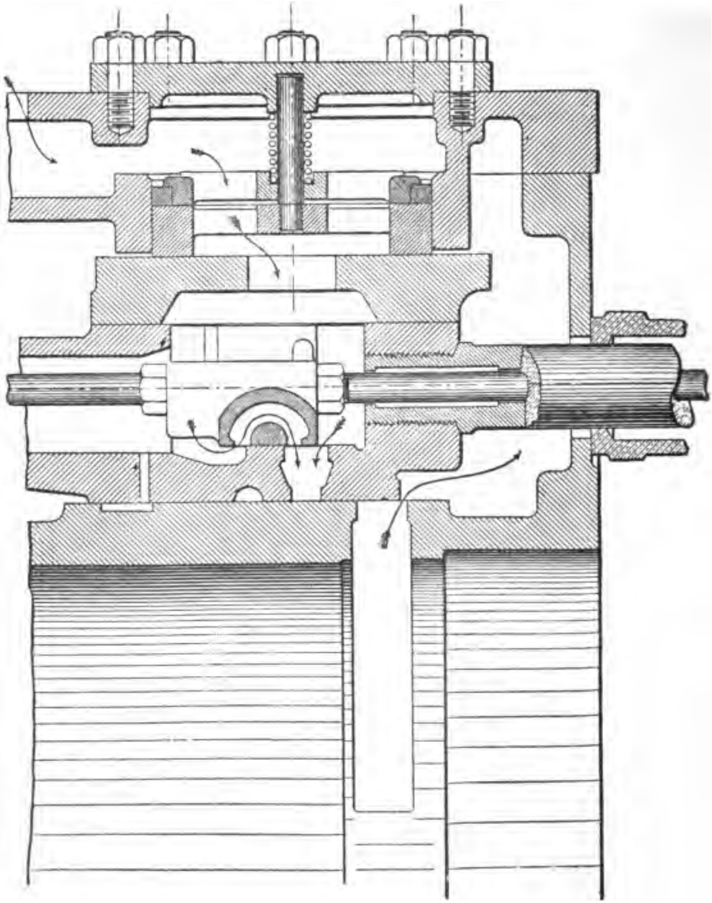


Fig. 124.

Corliss Valve Gears.—The principal Corliss gears may be divided into two parts, viz., those that operate the steam and exhaust valves by one eccentric, and those that use two eccentrics. In the former, the cut-off cannot be much later than one-third of the stroke; while in the latter it may be anywhere, but is usually not later than 80 per cent of the stroke. A Corliss cylinder is shown in section in fig. 128*, and in side elevation in fig. 129.* The steam valves are A, the exhaust B; both oscillate, but the former are brought back to effect cut-off by the springs C, C, when the jaws D, D are opened by the crosspiece *i*, fig. 130, and the part *e e* to which the end of the valve lever is connected is released. The inclination of *i* depends on the position of the levers *l*, which are controlled by the governor, so that the cut-off may be altered. The eccentric rod F operates the wrist plate L, and there may be two wrist plates and two eccentrics if desired. The following theoretical treatment of the subject is summarised from articles by Mr. James Dunlop, published in *The Practical Engineer* of June 24th, 1892.

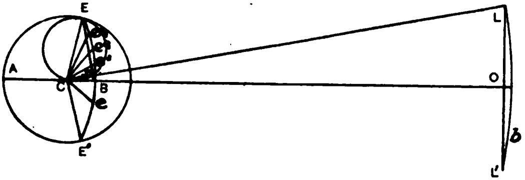


Fig. 125.

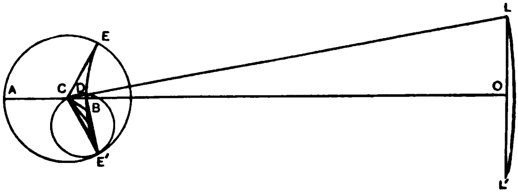


Fig. 126.

The motion of the valve is supposed harmonic—*i.e.*, the same as would be produced by a crank and very long connecting rod, and in fig. 131 A B is equal to the travel of the

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* From Holmes' "Steam Engine."

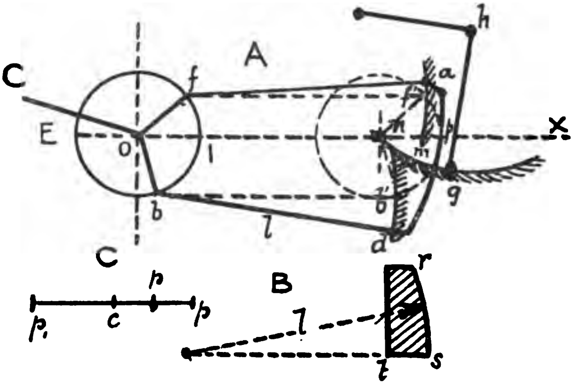

FIG. 127.

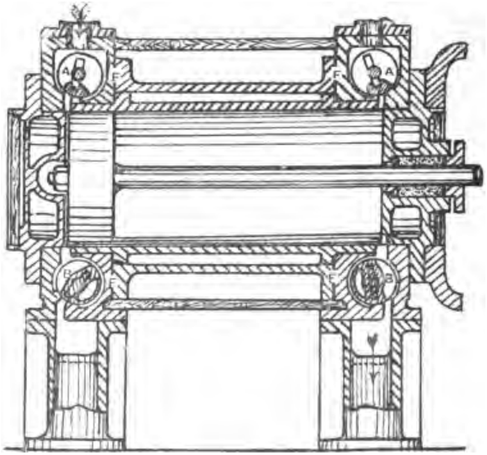

FIG. 128.

valve, and, to a smaller scale, the line of stroke; the direction of rotation is anti-clockwise, and M O + O L is the overlap of the valve when it has been drawn over the port by the spring. This is equivalent to what is generally termed the outside lap plus half the valve travel, so that chords of the circle O E drawn through O represent for each crank position the distance of the valve from the commencement of the stroke, less O M. If we draw the line O F through the points of intersection of the circle, whose radius is O L, and that whose diameter is O E, we have the crank at admission, and if L J is the width of port, it is full open when the crank is at O G, so that J B is the part of the stroke traversed before the port is fully opened. If there were no tripping, the cut-off would be at O P; but as tripping must occur before the

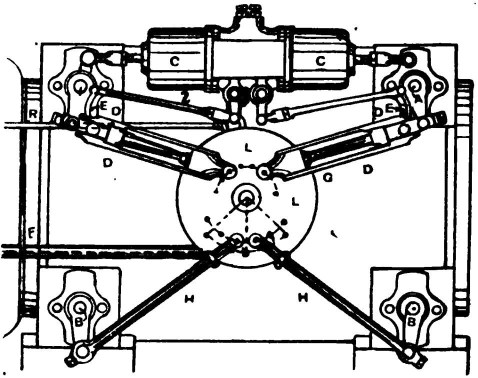

FIG. 129.

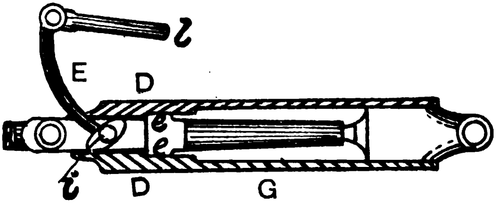

FIG. 130.

valve reaches the end of its stroke, the latest cut-off must be when the crank is at O E or the piston at M.

Now, much of the advantage of automatic expansion over throttling lies in the fact that engines can be "over-loaded"—*i.e.*, work at a less expansion, and therefore higher power, than that which is most economical, and for which the cylinder has been designed. Unnecessarily idle plant is thereby saved, with consequent reduction of capital expense. Hence an arrangement in which two eccentrics are used, so that the angle of advance of the eccentric that drives the steam valves need not be fixed to suit the exhaust valves, is preferable. The angle of advance of the exhaust eccentric may be made to suit the points of exhaust and compression, while the steam eccentric should lead the crank by an angle equal to A O M or E O B, fig. 132, which angle is fixed by the latest cut-off we wish to have. Thus, in the figure, A K is eight-tenths of A B, so that the latest cut-off is at eight-tenths stroke, rotation being clockways. On O M describe a circle, and draw O F, the crank at admission; this determines A N, which is the overlap of the steam valve

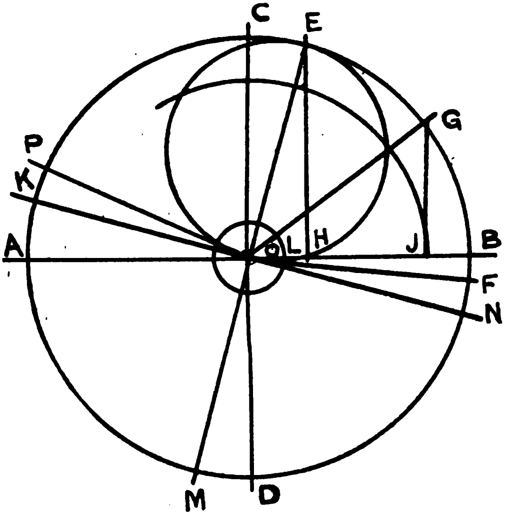

FIG. 131.

when pulled back by the spring; for it will be clearly seen that the parts of the radii intercepted between the circles M O and M E B are distances the valve has moved from the end of the stroke for any crank position. The port opening is the part of the diameter of the circle lying between the circle N T V and the circle O M. Thus, at the crank position O G the port opening is T H, where H is on T O produced. Of course this cannot be greater than the breadth of the port. The valve reaches its extreme travel when the crank is at O E, and tripping cannot be later than this. In designing the valve gear we may assume the angles of admission and cut-off, draw a circle with any radius O A, and thence find what A N and the width of port would be with the half travel O A, assuming, of course, some crank position for full port opening. Then, for the desired port

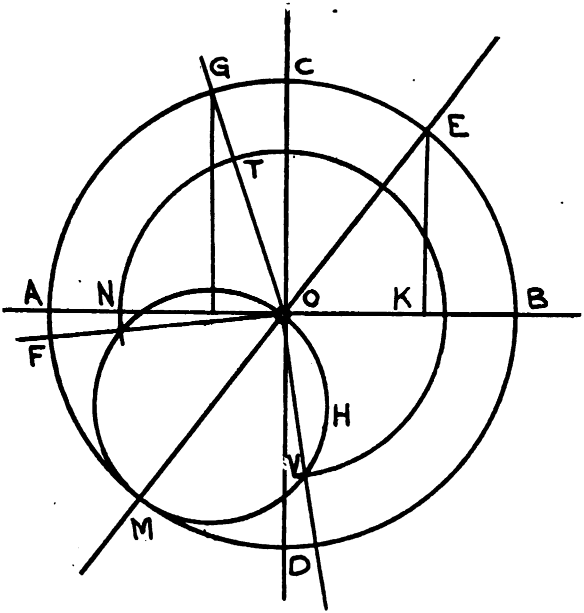

FIG. 132.

opening, we can find the travel by simple proportion, and also the actual length of A N.

Hackworth and Bremme's Valve Gears.—In fig. 133 are shown the above valve gears. In Hackworth's the valve rod, which at its upper end is connected to the valve spindle, is H E, and in Bremme's it is D R. In the former case the centre line of the eccentric A B is opposite the crank, while in the latter it is on the same side. The point F can have any position on the arc F G struck with centre C, but for a given ratio of expansion is a fixed point. The further F is from C K, the later is the cut-off, and the engine is reversed by shifting F C to G C. This rod is called the radius rod, and if to the left of C K the point C swings in the arc L M, and E and D trace out oval curves, the vertical displacement of whose points above and below the line A C, which is at right angles to the line of stroke, give the upward and downward displacements of the valve from its middle

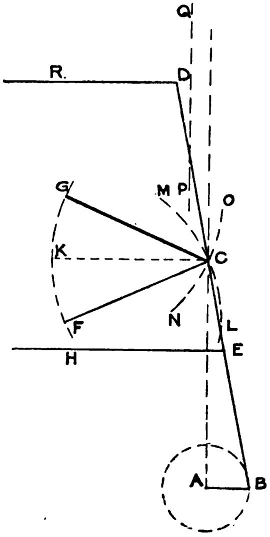

FIG. 133.

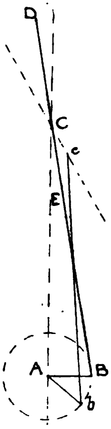

FIG. 134.

position; so that if P Q be drawn parallel to A C, and at a height above it equal to the outside lap at the lower end of the valve, then the height of D above P Q gives the port opening at the lower port. It can readily be seen that for any position of F C we can find the points of cut-off, exhaust, compression, and admission. The proportions given by the inventor for Bremme's valve gear are as follow, if the length A B is r:—

The eccentric rod B C is $6\,r$.

The radius rod F C is $6\,r$.

The lead arm D C is $4{\cdot}5\,r$.

The deviation angle F C K should not exceed 25 deg.

The outside lap is $\cdot 6\,r$, and the inside is negative, and $\cdot 0071\,r$. In the position of F C shown, the crank will rotate in the direction of the clock if the valve is moved by E, and in the opposite direction if moved by D.

The curvature of the path of C, and the obliquities of B C, H E, and R D, have considerable influence on the motion of the valve.

We shall first consider the case in which the path of C is a straight line perpendicular to F C, whose inclination to the vertical we will call α, fig. 134, and that B C is so long that the horizontal displacement of E is the same as the horizontal displacement of B.

Let θ be the angle that A b makes with A B, and let A B $= r$. Let x be the horizontal displacement of b and of e. Then

$$x = r \sin \theta.$$

Let y be the vertical displacement of e, and, therefore, the distance of the valve from its mid-stroke.

$$y = m\,x + \frac{1}{n}(r \cos \theta - m\,x),$$

where m is tangent of the angle c C A, and

$$n = \frac{\mathrm{B\,C}}{\mathrm{E\,C}}.$$

$$y = r\,q \sin(\theta + \alpha),$$

where $$q \cos \alpha = m\,\frac{n-1}{n}$$

$$q \sin \alpha = \frac{1}{n}.$$

$$\therefore q = \sqrt{\frac{m^2(n-1)^2 + 1}{n^2}}$$

and the valve is moved exactly as it would be if it were driven by an eccentric of throw $r . q$, whose angle of advance is α. When θ is zero or π,

$$y = \pm\, r\, q \sin \alpha . = \pm \frac{r}{n}.$$

so that the lead is constant; and as

$$r\, q = \frac{r}{n} \operatorname{cosec} \alpha,$$

the action of the gear in altering the grade of expansion is similar to that given by the movement of the equivalent eccentric at right angles to the crank.

In the case of Bremme's valve gear, we have

$$x = r \sin \theta$$

$$y = m\, r \sin \theta - p\, (r \cos \theta . - m\, x)$$

where

$$p = \frac{\mathrm{C\,D}}{\mathrm{B\,C}}.$$

$$y = r\, q \sin (\theta + \alpha)$$

where

$$q \cos \alpha = m\, (1 + p).$$

$$q \sin \alpha = - p.$$

$$\therefore q = \sqrt{p^2 + m^2\, (1 + p)^2}$$

so that this gear, also, is similar to the preceding case.

The subject of radial valve gears, however, is extremely complex when the effects due to the obliquity of the rods and the curvature of C's path are taken into account. These are thoroughly discussed in a paper by Mr. Joseph Harrison, published in the Proceedings of the Institution of Civil Engineers.

Radial Valve Gears.

In the design of valve gearing it is desirable, as far as possible, to secure conditions giving similar distribution of steam on both sides of the piston. Owing to the shortness of the connecting rod, the motion of the piston is not symmetrical in the advance and the return strokes; in the former—namely, that in which the movement is towards the crank shaft—the piston is more or less in advance of its harmonic position; that is, the one corresponding to an indefinitely long connecting rod, while in the latter the piston is behind such position.

If the valve is driven by an ordinary eccentric and rod, the obliquity of the latter is usually so small as scarcely to affect the harmonic motion of the valve. In an engine working under these conditions the result is, that if the valve is set with equal leads or port openings on the two sides, the point of cut-off is different in the two strokes, being later, and therefore causing a greater average pressure in the advance stroke. In the inverted vertical engine, the inequality in the acting forces arising from this cause is augmented on account of the weight of the reciprocating parts, and of the loss of area due to the piston rod, so that in marine engines it is common to give more lead at the lower than at the upper edge of the valve; but this unsymmetrical setting can be employed to an extent which is only partially effective in equalising the cut-off, for it introduces an inequality in the maximum port opening, as well as in the point at which steam is admitted to the two sides of the piston.

When a link motion or radial gear is used, irregularities in the motion of the valve are introduced, which may either increase or diminish the evil effects due to the shortness of the connecting rod, and, under favourable conditions, may afford an opportunity of obtaining partial or complete compensation.

In this paper, the nature of the required compensation is first examined, and then an analysis given of the motion of the valve in several typical forms of radial gears. The investigation shows how far the respective gears are capable of giving symmetrical steam distribution, and rules for their design are deduced. A method of determining the velocity of the valve at any point is also shown.

The valve diagram of Reuleaux, as modified by Grashof and by Coste and Maniquet,* is the one best suited to the purpose, and will now be briefly explained.

Fig. 135 is a diagram of the mechanism of a horizontal engine where C K, K L represent respectively the crank and connecting rod, and C P, P Q the eccentric radius and eccentric rod.

Since the motions of the piston and valve are similar to those of L and Q, it is seen that the displacements for mid-position, for any crank angle θ, are equal to the distances, measured parallel to A C, of K and P respectively from the circular arcs $b\,b'$, $d\,d'$, struck with radii equal to L K, Q P respectively.

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* Traité théorique et pratique des Machines à Vapeur au point de vue de la distribution. 1886.

In Reuleaux's valve diagram, fig. 136, a circle of reference is taken, representing the path of both crank pin and eccentric centre, each to its proper scale. C A is the position of the crank at the beginning of the advance stroke, the direction of rotation being from A towards K. C D is a line set back from C A an amount equal to α, the angular advance of the eccentric. The radii R and R′ for the arcs $b\,b'$, $d\,d'$ are respectively $R = \mu \cdot CA$ and $R' = \nu \cdot CA$, μ and ν being the ratios

$$\frac{\text{connecting rod}}{\text{crank}} \text{ and } \frac{\text{eccentric rod}}{\text{eccentric radius}}.$$

Suppose, now, the crank to have turned through an angle θ, and to be in the position C K; then, comparing figs. 135 and 136, it is seen that K *m* is the displacement of the piston from its mid-position, and K *n* that of the valve; if the connecting and eccentric rods were of infinite length, the corresponding displacements would be K M and K N; the deviations from harmonic motion for the position K are therefore equal respectively to M *m* and N *n*. It will be convenient to refer to the harmonic displacement as primary, and to the deviation as secondary.

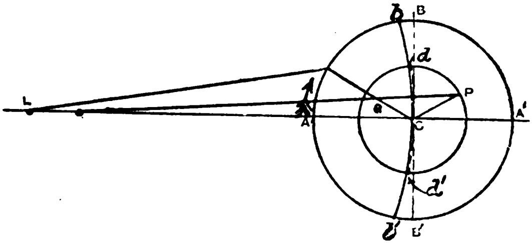


FIG. 135.

Expressed analytically, putting *k* for the length of the crank, and ρ for the eccentric radius—

$$\text{primary displacement} = KM = k\cos\theta \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\text{secondary displacement} = -\,Mm = -\frac{CM^2}{2R - Mm}$$

$$= -\frac{CM^2}{2R} \text{ nearly} = -\frac{k}{2\mu}\sin^2\theta \quad . \quad . \quad . \quad . \quad (2)$$

the approximation being equivalent to substituting for the circular arc $b\,b'$ a parabola having the same radius of curvature at C. The actual displacement of the piston is then

$$k \cos \theta - \frac{k}{2\mu} \sin^2 \theta.$$

For the valve the corresponding expressions are—

$$\text{primary displacement} = \Delta = \mathrm{K\,N} = \rho \sin(\theta + a) \quad . \quad (3)$$

$$\text{secondary displacement} = \delta = \mathrm{N}\,n = \frac{\rho}{2\nu} \cos^2 (\theta + a) \quad . \quad (4)$$

the actual displacement being the sum of the two.

In order to illustrate the relative magnitudes of the quantities dealt with, it may be stated that for a connecting rod four cranks long, B b = 0·1270 k—*i.e.*, the greatest value of the deviation of the piston is 12·7 per cent of the crank length; and the maximum error introduced into the calculation of this deviation by the substitution of a parabola for the circular arc $b\,b'$ is about 1½ per cent of B b, or only ⅕ per cent of the length of the crank.

Fig. 137 shows Reuleaux's diagram, set out so as to give the port opening K i for any crank-pin position K; $s\,s'$, $s\,s'$ are circular arcs drawn with radius $R' = \nu \cdot$ C A, the same as that for $d\,d'$, and distant from the latter by the amounts C F, C G, equal respectively to the advance and the return laps. It will be convenient to call D D′ and $d\,d'$ the displacement line and displacement curve, and $s\,s'$ the steam curve.

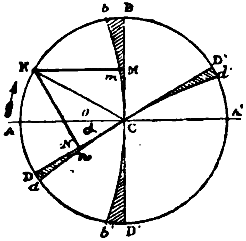


FIG. 136.

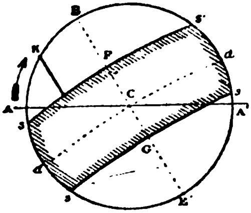


FIG. 137.

The effect of altering the length of the valve rod is equivalent to increasing the lap on one edge of the valve, and diminishing by an equal amount the lap on the other edge, and could be exhibited on the diagram by suitably shifting the arcs $s\,s'$, $s\,s'$. The effect of any play in the joints could also be shown by an alternate lateral shifting of $s\,s'$ when reversal of motion takes place.

The next thing considered is the problem of finding the motion of the valve required for symmetrical steam distribution with equal maximum port openings, and one solution occurs if the relative simultaneous displacements of piston and valve are at all points the same as if both had harmonic motion.

Let α be the angular advance for the primary motion of the valve, and D D′, fig. 138, the corresponding displacement line. If the motion of piston and valve were both harmonic, then for any crank-pin position K′ the displacements would be respectively K′ M′ and K′ N′; but owing to the shortness of the connecting rod, the crank pin is at K when the piston displacement is K′ M′, where K is taken such that K m = K′ M′; if, now, K n be made equal to K′ N′, N n will be the secondary displacement of the valve, and the locus of n the secondary displacement curve required. Expressed analytically—

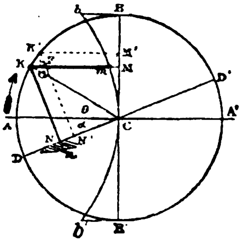


Fig. 138.

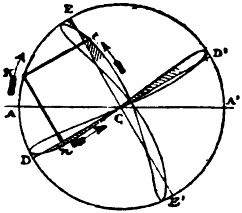


Fig. 139.

$$\mathrm{N}\,n = \mathrm{K'F} = \mathrm{K\,K'}\cos\mathrm{K\,K'\,F} = \frac{\mathrm{KG}}{\sin\theta}\cos(\alpha+\theta)\text{ nearly}$$

$$= \mathrm{M}\,m\,\frac{\cos(\alpha+\theta)}{\sin\theta} = \frac{\mathrm{CM}^2}{2\,\mathrm{R}}\;\frac{\cos(\alpha+\theta)}{\sin\theta}\text{ nearly}$$

$$= \frac{\mathrm{C\,K}^2\sin^2\theta\cos(\alpha+\theta)}{2\,\mu\cdot\mathrm{C\,A}\cdot\sin\theta}$$

or $$\delta = \frac{\rho}{2\,\mu}\sin\theta\cos(\alpha+\theta)\quad\ldots\ldots\ldots\ldots\quad(5)$$

where, as before, θ is the crank angle, ρ the radius of the eccentric, and μ the ratio $\dfrac{\text{connecting rod}}{\text{crank}}$.

This curve is set out to scale in fig. 139, for the case in which $\mu = 4$ and $\alpha = 30$ deg., as is also the corresponding velocity curve, to be explained subsequently.

It will be of advantage to consider separately the component eccentrics a and b, obtained by resolving ρ in the direction of the crank produced backwards, and perpendicular thereto, the respective values being $a = \rho \sin \alpha$ and $b = \rho \cos \alpha$. The displacement Δ due to ρ is then equal to the algebraical sum of the displacements X and Y, corresponding to a and b, as is seen from the equation—

$$\Delta = \rho \sin (\alpha + \theta) = a \cos \theta + b \sin \theta \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The compensating secondary displacements x and y for a and b can be found by the method previously given, or obtained from equation (5) by putting $\alpha = 90$ deg., $\rho = a$ for the former, and $\alpha = 0$, $\rho = b$ for the latter; thus—

$$x = -\frac{a}{2\mu} \sin^2 \theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$y = \frac{b}{4\mu} \sin 2\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

and it is easily seen that $\delta = x + y$.

These curves are drawn to scale in fig. 140, for the data $\mu = 4$, $\alpha = 30$ deg. In the figure the same reference circle is taken for both a and b, requiring different scales for X and Y, but the secondary displacements x and y are set out both to the same scale, viz., that on which the radius of the reference circle represents ρ, this being convenient for summation and for comparison with fig. 139. The curve for x is a parabola, concave towards A; that for y a looped figure, closely related to the parabola, as will presently appear.

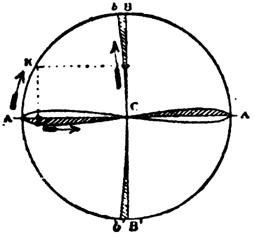


FIG. 140.

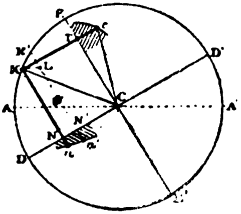


FIG. 141.

The velocity of the valve, which it is important to take notice of, can be readily indicated on the diagram.

Suppose the motion of the valve known and the displacement curve drawn. Take two adjacent positions K, K′, fig. 141, and draw K L parallel to $n\,n'$; then K′ L, the difference between K′ n' and K n, is equal to the movement of the valve whilst K moves to K′, or—

$$\frac{\text{velocity of valve}}{\text{velocity of K}} = \frac{\text{K}'\text{L}}{\text{K K}'} = \frac{\text{K}\,t}{\text{C K}},$$

where K t is drawn perpendicular to K′ L, and C t to K L, or in the limit to the curve at n. Putting ω for the angular velocity of the crank shaft, the above equation may be written—

$$\text{velocity of valve} = \frac{\text{velocity of K}}{\text{C K}}\,.\,\text{K}\,t = \omega\,.\,\text{K}\,t.$$

The line E E′ drawn perpendicular to D D′ may be called the velocity line, and the locus of t the velocity curve. K T, which is at right angles to E E′, represents the velocity of the valve for harmonic motion, T t being the alteration due to the secondary displacement. Expressly analytically—

$$\left.\begin{array}{l}\text{primary velocity} = \text{V} = \omega\,.\,\text{KT} = \omega\,.\,\rho\,.\cos(\alpha + \theta)\\ \text{secondary velocity} = v = \omega\,.\,\text{T}\,t\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\end{array}\right\}(9)$$

In particular it is seen that, with harmonic motion, the velocity of the valve at cut-off is equal to the angular velocity of the crank shaft multiplied by half the length of the steam line S S′, fig. 150.

The method just explained is now applied to ascertain how the velocity of the valve is influenced by the secondary displacements previously considered, which give symmetrical steam distribution. The velocity curve is shown in fig. 139, being determined graphically from the displacement curve in the way explained. Its equation is obtained as follows:

From equation (5) N $n = \frac{\rho}{2\mu}\sin\theta\cos(\theta + \alpha)$, also C N $=$ $\rho\cos(\theta + \alpha)$; therefore (fig. 141) $\frac{\text{T}\,t}{\text{C T}} = \tan \text{T}\,c\,t =$ tangent of the angle which the curve at n makes with C N $=$

$$\frac{d\,(\text{N}\,n)}{-\,d\,(\text{C N})} = \frac{\cos(2\,\theta + \alpha)}{2\,\mu\sin(\theta + \alpha)};$$

also $CT = \rho \sin(\theta + \alpha)$, from which $Tt = \frac{\rho}{2\mu} \cos(2\theta + \alpha)$; and the equation to the secondary velocity curve is—

$$v = \omega . Tt = \omega . \frac{\rho}{2\mu} \cos(2\theta + \alpha) \quad . \quad . \quad . \quad . \quad (10)$$

Expanding the right-hand side, this may be written—

$$v = \omega \frac{\rho}{2\mu} (\cos\alpha \cos 2\theta - \sin\alpha \sin 2\theta)$$

$$= - \frac{\omega a}{2\mu} \sin 2\theta + \frac{\omega b}{2\mu} \cos 2\theta . = v_y + v_x$$

having regard to sign, where

$$v_y = - \frac{\omega a}{2\mu} \sin 2\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

$$v_x = \frac{\omega b}{2\mu} \cos 2\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

v_y and v_x being the secondary velocities corresponding respectively to the deviations for the component eccentrics a and b. The values of v_y and v_x are set out in fig. 142, and it will be seen that the looped curve and the parabola again present themselves.

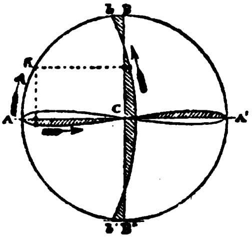


FIG. 142.

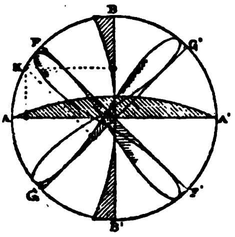


FIG. 143.

From an inspection of fig. 139 the influence of the secondary displacement in modifying the velocity of the valve can be traced. It is observed that the effect is opposite in the two strokes, a quicker cut-off, or admission, in the advance stroke, implying a correspondingly slower one in the return, so that, on the whole cycle, no advantage is obtained in this respect.

All the expressions hitherto obtained for δ and v will be found included in the general formula $A \, . \sin (2 \theta + \psi) + B$, suitable values being given to A, B, and ψ. Figs. 143 and 144 have been drawn to illustrate the geometrical properties of the curves. In fig. 143 the case where $\delta \propto \sin 2^2 \theta$, as in equation (7), is set out on the base lines A A′, B B′, and on two others inclined at 45 deg. to them; and in fig. 144 the same thing has been done when $\delta \propto \sin 2 \theta$, as in equation (8). These figures show the close relations and reciprocal properties of the looped and parabolic curves, and also suggest, as will afterwards appear, that the disturbance, caused by the obliquity of an oscillating rod, may be balanced by that due to a rod at right angles to it or inclined in some other direction.

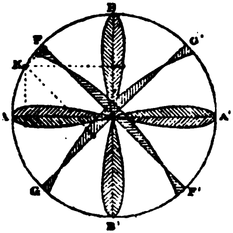


FIG. 144.

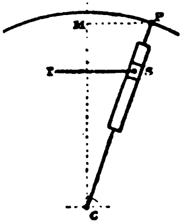


FIG. 145.

Fig. 145 further illustrates the subject. C P is a rod, centred at C and rotating uniformly, or oscillating about C M in such a manner that P M varies harmonically. S is a block, sliding in the groove, with harmonic motion relative to C P, of the same period as that of C P, and imparting motion through S T to a valve, or other reciprocating piece. Assuming S T to be of considerable length, so that the effect of its obliquity may be neglected, it is easily seen that under these conditions a deviation $\delta = A \sin (2 \vartheta + \psi) + B$ is given to the reciprocating piece. In particular it is to be noted that when the phases of S and C P agree, $\delta \propto \sin^2 \theta$; and that when they differ by 90 deg., $\delta \propto \sin 2 \theta$. An example of the latter is seen in Morton's valve gear.

If S T drive a valve, the elements of the motion of S, in the groove for complete compensation, can be obtained from equation (5) by suitable transposition.

The next problem examined is that of the motion of an intermediate point in a connecting rod, or other similarly moving piece, which motion is often utilised in actuating the slide valve. In fig. 146 S is the point taken, the path of which is drawn, and is seen to be approximately an ellipse. Considering separately the vertical and the horizontal displacements of S from mean position, for the former

$$SM_2 = PN \times \frac{SQ}{PQ},$$

which, being proportional to P N, is harmonic ; and for the latter

$$SM_2 = PM - GH,$$

which is approximately harmonic, the deviation G H being relatively small when the obliquity of P Q is not excessive. Taking A B A′ as the reference circle for the component displacements of S, then P N, measured on a suitable scale, will represent S N_2, and P m is made equal to S M_2. The locus of m is an elliptic arc, since

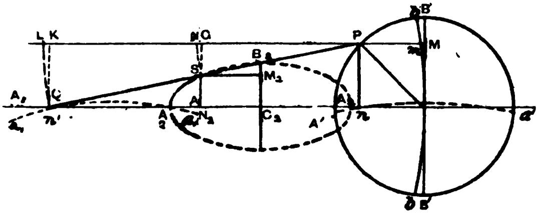


FIG. 146.

$$Mm = GH = \frac{PS}{PQ} . KL ;$$

but it may be drawn with sufficient accuracy as a circular arc having a radius equal to

$$PQ \times \frac{PQ}{PS};$$

or, expressed analytically, by the parabolic approximation,

$$Mm = -\frac{PS}{PQ} . \frac{CP}{2\mu} . \sin^2\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

All the three curves have the same radius of curvature at C.

If the point Q be guided by a radius rod, and move in a flat circular arc a_1, a'_1, radius r, instead of in a straight line, the vertical displacement of S will be affected by the amount

$$\frac{\text{P S}}{\text{Q P}} \cdot \text{Q}\, n, \text{ very nearly,}$$

and this is approximately represented on the diagram by the arc $a\ a'$, similarly placed, but struck with a radius

$$r \times \frac{\text{Q P}}{\text{P S}},$$

the scale for M n then being the same as that for M m. The horizontal displacement of S will also be altered; but, except when the obliquity of P Q is great, the alteration may be considered as a small quantity of an order higher than secondary, and therefore is not taken into account.

Coming now to the more immediate object of this investigation, it is known that in a valve driven by radial gear the deviation from harmonic motion is not great. An accurate analysis is impracticable, if not impossible, on account of the complexity of the equations thereby introduced; and the following approximate method is adopted, based on the principle of the superposition of small quantities: First, the elements of the primary displacement are found on the assumption that the vibrating rods are very long; next, the secondary displacements are estimated, each one separately, as if it alone existed for the time, and these are added algebraically. With the proportions of the parts usually met with, the results so obtained are sufficiently accurate for practical purposes, without going to a third approximation, and will be found of substantial assistance in the design of valve gears.

HACKWORTH-MARSHALL GEAR.

The type of radial gear which naturally comes first under consideration is the Hackworth, as modified by Marshall, and shown diagrammatically in its two forms in figs. 147 and 148. C P is an eccentric set in line with the crank; Q is constrained to move along the arc F G, either by a radius rod or slide; alteration of the cut-off and reversal of the engine are effected by turning F G about O, O being the position of Q when the engine is on either dead centre. It is necessary to distinguish between forward and backward rotation, and for the present purpose it is sufficient to define the former, shown by the arrows, as that in which, during the advance stroke, the crank pin is on the same side as the valve.

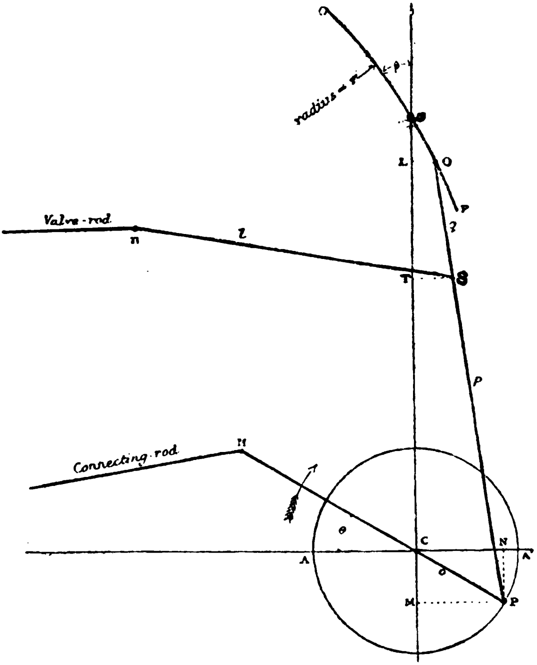


FIG. 147.—Hackworth-Marshall Gear. Form I.

The following symbols, some of which have been previously used, are placed here for convenience :—

k = radius of crank.
μ = ratio connecting rod to crank.
c = radius of eccentric.
θ = angle of crank from initial position C A.
ϕ = angle of connecting rod.
ω = angular velocity of crank shaft.
s = length of P Q.
p = length of P S.

q = length of Q S.
l = length of radius rod S H.
r = radius of F G.
β = inclination, at O, of slide F G to C O.
ρ = radius of virtual eccentric for the primary displacement of the valve.
α = angle of advance of virtual eccentric ρ.
$a = \rho \sin \alpha$ = component of ρ parallel to crank.
$b = \rho \cos \alpha$ = component of ρ perpendicular to crank.

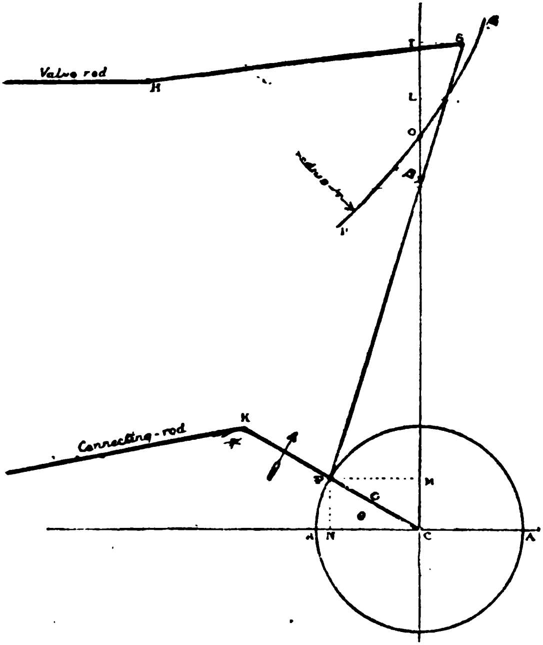


Fig. 148.—Hackworth-Marshall Gear. Form II.

Primary Displacement.—Referring to fig. 147, it is seen that

$$\mathrm{S\,T} = \frac{\mathrm{S\,Q}}{\mathrm{P\,Q}} \cdot \mathrm{P\,M} + \frac{\mathrm{S\,P}}{\mathrm{P\,Q}} \cdot \mathrm{Q\,L}. \quad . \quad . \quad . \quad . \qquad (14)$$

Now, $PM = c \cos\theta$, and also, when the rods are all very long relative to CP, $QL = OL \tan\beta = PN \tan\beta = c \sin\theta \tan\beta$, so that, for the primary displacement, substituting in (14)

$$\Delta = \frac{q}{s} . c . \cos\theta + \frac{p}{s} . c . \tan\beta \sin\theta \quad . \quad . \quad . \quad . \quad (15)$$

comparing with which equation (6), the elements of the virtual eccentric are found, viz. :

$$a = \frac{q}{s} . c \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

$$b = \frac{p}{s} c \tan \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

$$\rho \sqrt{a^2 + b^2} = \frac{c}{s} \sqrt{q^2 + p^2 \tan^2\beta} \quad . \quad . \quad (18)$$

also $$\tan\alpha = \frac{a}{b} = \frac{q}{p} \cot\beta,$$

or $$\tan\alpha . \tan = \frac{q}{p} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

From these equations it follows that a, which equals lap + lead, is constant, so that the lead is also constant, and, further, that b is proportional to $\tan\beta$; the locus of the virtual eccentric centre is therefore a straight line, perpendicular to the crank, as shown in fig. 149; this property is common to a great many radial and other gears, and Reuleaux's diagram for the case is conveniently set out thus :—

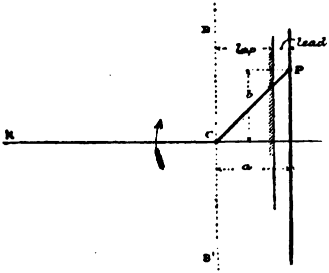


Fig. 149.

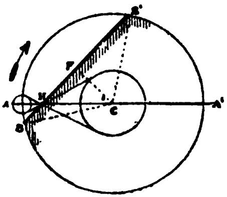


Fig. 150.

Take any reference circle, fig. 150, and, with C and A as centres, draw circles with radii respectively proportional to the lap and lead, and find H, the intersection of common tangents to these circles; the steam line S S′, for any setting of the gear corresponding to C P, fig. 149, is then drawn through H, in direction such that the angle A H S is equal to B C P, fig. 149, the angular advance of the virtual eccentric.

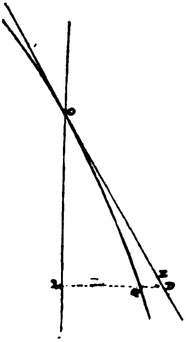


FIG. 151.

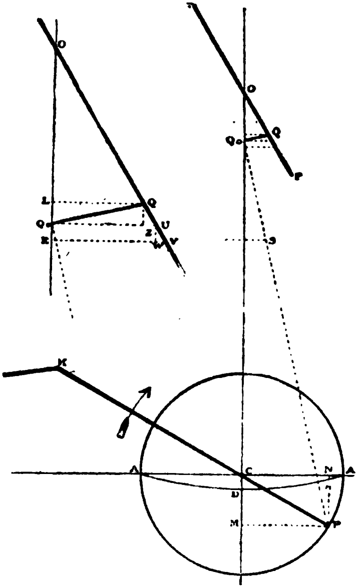


FIG. 152.

The velocity of the valve at admission or cut-off, as previously shown, equals $\omega \cdot \frac{SS'}{2} = \omega \times CF \cdot \tan SCF = \omega \times$ lap $\times \tan \frac{SCS'}{2}$, and so, for any setting of the gear, is proportional to the tangent of half the angle subtended by the steam line at the centre of the reference circle. The same reference circle being used for all settings, the scale must vary, and is readily found for any position, since C F, the perpendicular on the steam line, represents the lap.

Secondary Displacements.—(1) Considering first the link S H, fig. 147, and assuming all the other rods to be very long, the secondary displacement due to the oscillations of this link is

$$\delta_1 = \frac{\mathrm{P\,N}^2}{2\,\mathrm{S\,H}} = \frac{c^2}{2\,l}\sin^2\theta. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (20)$$

(2) In estimating next the effect of the radius rod, r or curvature of F G, as compared with a straight slide, the usual assumption is made as to length of the remaining rods; then, fig. 151, $\mathrm{Q\,D} = \mathrm{Q\,E}\sec\beta = \frac{\mathrm{O\,E}^2}{2\,r}\sec\beta = \frac{(\mathrm{O\,L}\sec\beta)^2}{2\,r}$ $\sec\beta$ nearly $= \frac{c^2\sin^2\theta}{2\,r}\sec^3\beta$, and

$$\delta_2 = -\frac{p}{s}\,\mathrm{Q\,D} = -\frac{p\,c^2\sec^3\beta}{2\,r\,s}\sin^2\theta \quad . \quad . \qquad (21)$$

(3) and (4) The last disturbance to be examined is that due to the obliquity of the rod P Q, l and r being supposed indefinitely long.

In fig. 152, Q Q_o is a circular arc, centre P, radius P Q or s, so that Q_o is the position of Q when $\beta = 0$. Take $\mathrm{O\,E} = \mathrm{P\,N}$, and draw E V and the other lines as shown. Also with centre O, radius s, draw the arc A D A′. Then the deviation from harmonic motion due to the obliquity of P Q $= \frac{p}{s}$ $(\mathrm{L\,Q} - \mathrm{E\,V}) = \delta_3 + \delta_4$, say, where $\delta_3 = -\frac{p}{s}\,\mathrm{V\,W}$, $\delta_4 = -\frac{p}{s}\,\mathrm{U\,Z}$.

Now, $\mathrm{V\,W} = \mathrm{W\,U}\tan\beta = \mathrm{Q}_o\,\mathrm{E}\tan\beta = \mathrm{N}\,n\tan\beta = \frac{c^2}{2\,s}$ $(1 - \cos^2\theta)\tan\beta = \frac{c^2}{2\,s}\tan\beta\sin^2\theta$; and $\mathrm{U\,Z} = \mathrm{Q\,Z}\tan$ $\beta = \mathrm{Q\,Q}_o\,.\,\frac{\mathrm{P\,M}}{\mathrm{P\,Q}_o}\tan\beta = \frac{\mathrm{E\,V}\,.\,\mathrm{P\,M}}{s}\tan\beta$ nearly $= \frac{\mathrm{P\,N}\,.\,\mathrm{P\,M}}{s}$ $\tan^2\beta = \frac{c^2}{2\,s}\tan^2\beta\sin 2\,\theta$; inserting these values in the expressions for δ_3 and δ_4,

$$\delta_3 = -\frac{p\,c^2\tan\beta}{2\,s^2}\sin^2\theta = -\frac{a\,b}{2\,q}\sin^2\theta \quad . \quad . \quad . \qquad (22)$$

$$\delta_4 = -\frac{p\,c^2\tan^2\beta}{2\,s^2}\sin 2\,\theta = -\frac{b^2}{2\,p}\sin 2\,\theta \quad . \quad . \qquad (23)$$

Reviewing these results, it is seen that δ_1 is independent of β, and δ_2 nearly so, unless β be exceptionally large, so that a relation between l and r can be found, for which δ_1 and δ_2 nearly neutralise each other for all settings of the gear. The analysis, however, shows that the proportions may be arranged so that δ_1 and δ_2 have a resultant equal to x, equation (7), where x is one component of the compensation required on account of the obliquity of the connecting rod.

This result is secured when δ_1 + average value of $\delta_2 = x$, or

$$\frac{s}{l} + \frac{q}{c} \cdot \frac{1}{\mu} = \frac{p}{r} \cdot \frac{1 + \sec^3 \beta}{2} \quad . \quad . \quad . \quad . \quad . \quad (24)$$

an equation applicable without change to both forms of the gear, and from which the relation between l and r is to be calculated.

It should be observed that the equations refer to forward rotation of the first form of the gear, fig. 147, and that corresponding values of the second form, fig. 148, can be deduced by changing the signs of c, q, and β. It is also to be remembered that for backward rotation θ, β and b are negative.

The deviation δ_3, due to the unsymmetrical oscillation of Q on each side of O, can be balanced for any one value of β, but not for a range of values, and to effect the former the relation between l and r is to be found from the equation

$$\delta_1 + \delta_2 + \delta_3 = x,$$

or
$$\frac{s}{l} + \frac{q}{c} \cdot \frac{1}{\mu} = \frac{p}{r} \sec^3 \beta + \frac{p}{s} \tan \beta \quad . \quad . \quad . \quad (25)$$

the sign of the last term being changed when referring to Form II. of the gear. This modification has the effect of increasing the error δ_3 if the engine be reversed, and so is only applicable to the case where reversal is but occasional. If the engine is required to run equally in the two directions, it will be necessary to revert to equation (24), and leave δ_3 unbalanced.

Lastly, δ_4, proportional to $\sin 2\theta$, is advantageous when it tends to the value y, equation (8); and $\delta_4 = y$ when

$$\frac{s}{c} = -2\mu \tan \beta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

for both forms of the gear. Interpreting this equation, it is seen that δ_4 tends to neutralise the inequality due to the connecting rod when the engine is running backwards; the reverse is the case when going forwards.

It appears from the analysis that the Hackworth-Marshall gear is capable of effecting a very equable steam distribution when designed for a special cut-off in backward rotation, but that it is deficient in this respect when used for forward rotation or for reversal.

JOY VALVE GEAR.

In this gear the eccentric is dispensed with, and the locus of the point P_2, fig. 153, takes the place of the circle described by the eccentric centre. The path of P_2 is shown, and if the obliquities of the rods are very small, the curve is an ellipse, with semi-axes respectively equal to

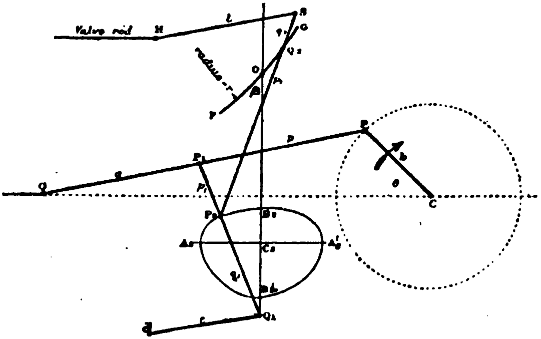


FIG. 153.

$$\frac{A_2 A'_2}{2} = \frac{Q_1 P_2}{P_1 Q_1} \cdot CP = \frac{q_1}{s_1} k, \text{ and } \frac{B_2 B'_2}{2} = \frac{Q P_1}{P Q} \cdot CP = \frac{q}{s} k,$$

the notation being sufficiently indicated on the figure.

Primary Displacement.—If the above values are substituted for c in equations (16) and (17), the components of the primary displacement are at once deduced, viz.:

$$a = \frac{A_2 A'_2}{2} \cdot \frac{Q_2 S}{P_2 Q_2} = \frac{q_1 q_2}{s_1 s_2} k. \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

$$b = \frac{B_2 B'_2}{2} \cdot \frac{P_2 S}{P_2 Q_2} \tan\beta = \frac{q\,p_2}{s\,s_2} k \tan\beta \quad . \quad . \quad (28)$$

from which

$$\tan \alpha \tan \beta = \frac{s\, q_1\, q_2}{q\, s_1\, p_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

and the locus of the virtual eccentric centre is a straight line, as in the Hackworth-Marshall gear.

Secondary Displacements.—These are estimated separately, as before.

(1) For the rod S H

$$\delta_1 = \left(\frac{B_2\,B'_2}{2}\right)^2 \cdot \frac{\sin^2\theta}{2\,l} = \frac{q^2\,k^2}{2\,s^2\,l}\sin^2\theta \quad . \quad . \quad (30)$$

(2) For the curvature of the slide F G

$$\delta_2 = -\left(\frac{B_2\,B'_2}{2}\right)^2 \frac{p_2 \cdot \sec^3\beta}{s_2\,2\,r}\sin^2\theta = -\frac{q^2\,p_2\,k^2 \sec^3\beta}{2\,s^2\,s_2\,r}\sin^2\theta \, (31)$$

(3) and (4) Obliquity of rod $P_2\,Q_2$. The effect of this can be found by a method analogous to that previously employed, or the result deduced by substituting respectively

$$\left(\frac{A_2\,A'_2}{2}\right)^2,$$

p_2, s_2, $-\tan\beta$ for c^2, p, s, $\tan\beta$, in equation (22), and

$$\left(\frac{A_2\,A'_2}{2}\right)\left(\frac{B_2\,B'_2}{2}\right),$$

p_2, s_2, for c^2, p, s in equation 23, whence—

$$\delta_3 = \frac{q^2{}_1\,p_2\,k^2 \tan\beta}{2\,s^2{}_1\,s^2{}_2}\sin^2\theta \quad . \quad . \quad . \quad (32)$$

$$\delta_4 = -\frac{q\,q_1\,p_2\,k^2 \tan^2\beta}{2\,s\,s_1\,s^2{}_2}\sin 2\,\theta \quad . \quad . \quad (33)$$

(5) Effect of obliquity of connecting rod. The point P_1 has a horizontal deviation from harmonic motion [compare equation (13)] equal to

$$\frac{p\,k^2}{2\,s^2}\sin^2\theta,$$

and this, in transmission through the rods $P_1\,Q_1$, $P_2\,Q_2$ to S is reduced in the ratio

$$\frac{P_1\,Q_1}{P_2\,Q_1} \times \frac{P_2\,Q_2}{S\,Q_2},$$

so that

$$\delta_5 = -\frac{p\,q_1\,q_2\,k^2}{2\,s^2\,s_1\,s_2}\sin^2\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

(6) Obliquity of rod $P_1 Q_1$. Comparing the displacement of P_2 from $A_2 A_2'$ with that of P_1 from $Q C$, the latter being harmonic, the deviation is found equal to

$$- \frac{p_1 k^2}{2 s_1^2} \sin^2 \theta$$

[see equation (13)], and in transmission to S this affects the valve by the amount

$$\delta_6 = - \frac{p_1 p_2 k^2 \tan \beta}{2 s_1^2 s_2} \sin^2 \theta \quad . \quad . \quad . \quad . \quad (35)$$

(7) Radius rod Q_1 J. The effect of this on the valve is usually very small, its value being

$$\delta_7 = \frac{q^2 p_1 q_2 k^2}{2 s^2 s_1 s_2 t} \sin^2 \theta \quad . \quad . \quad . \quad . \quad . \quad (36)$$

The secondary disturbances have now all been considered; comparing them one with another, and with equations (7) and (8), as in the Hackworth gear, the following equations are deduced from which to determine the relative proportions for best steam distribution :—

δ_1 + average value of $\delta_2 + \delta_5 + \delta_7 = x$, or

$$\frac{s_2}{q_2} \cdot \frac{1}{l} + \frac{q_1}{s_1} \cdot \frac{1}{q} + \frac{p_1}{s_1} \cdot \frac{1}{t} = \frac{p_2}{q_2} \cdot \frac{1 + \sec^3 \beta}{2 r.} \quad . \quad . \quad . \quad (37)$$

$\delta_3 + \delta_6 = 0$, or

$$q_1^2 = p_1 s_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

and $\delta_4 = y$, or

$$2 s q_1 \tan \beta + s_1 s_2 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

From equation (37), having previously determined on the other proportions, the radius of the slide is found which gives the best average value of the compensation x, for the whole range between full forward and full backward gear.

Equation (38) expresses the condition that Q_2 shall oscillate equally on one side of O, which in the Hackworth-Marshall gear is impossible of attainment, and would be impossible here if the rod $P_1 Q_1$ were dispensed with, and the end of $P_2 Q_2$ attached directly to the connecting rod. The function of the link $P_1 Q_1$ is, therefore, to permit of this adjustment, as well as to diminish the extent of the swing of $P_2 Q_2$.

From equation (39) it is inferred that the compensation y cannot be obtained except for one particular negative value of β, and the gear agrees with the Hackworth in this respect, working better when running backwards.

MORTON VALVE GEAR.

The analysis will be confined to one form of this gear, that shown in figs. 154 and 155, the latter giving the positions with the engine on the dead centres. R Q_2 is a radius rod centred at R, which, together with the attachment at P_1, causes Q_2 to oscillate in a circular arc; the point S vibrates on each side of this arc owing to the connection between P_2 and T, Q T forming one rigid piece with the crosshead; alteration of the cut-off or reversal are effected by shifting the position of the block L in the sector H, the latter being fixed to the end of the valve rod.

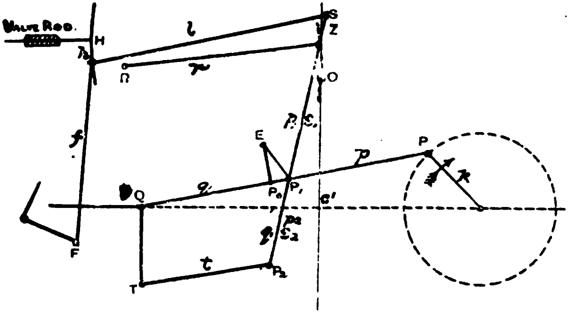


FIG. 154.

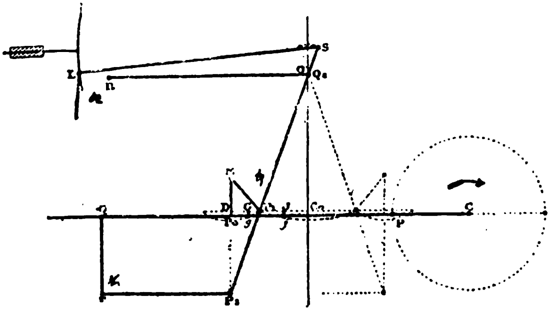


FIG. 155.

In fig. 154, put $P_1 S = p_1$, $P_2 S = p_2$, $P_1 Q_2 = s_1$, $P_2 Q_2 = s_2$, $P_1 P_2 = q_1$, $Q_2 S = q_2$, $E P_1 = e$, h = length of the tangent to the sector at L on the valve rod—*i.e.*, $h = HL$ nearly, $P P_0 = p$, $P Q = s$, &c., the rest of the notation being sufficiently clear from the figure.

Primary Displacement.—If C P and Q T be assumed relatively very short, the angular oscillations of the rods will then all be very small, and the conditions suitable for determining the elements of the primary displacement; in this case P_1 and P_2 both coincide very nearly with P_0, and the point S moves approximately in an elliptic path, shown enlarged in fig. 156, of which the semi-axes are

$$CA = \frac{q_2}{s_2} k, \quad CB = \frac{q}{s} k.$$

The motion of the valve is derived from that of S through the connecting link S L. In the figure draw through L a line parallel to the valve rod, and take $CO = SL = l$; then C O, inclined at an angle $\tan \frac{h}{l} = \beta$ say to C H, is the mean direction of the rod S L.

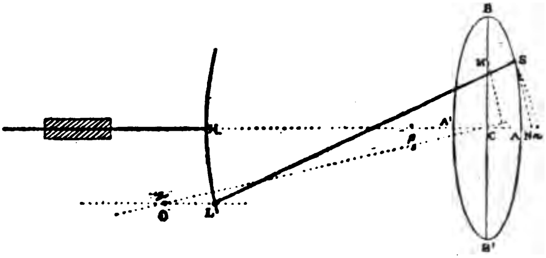


FIG. 156.

Draw S N perpendicular to O C; then, if the obliquity of S L to C O were very small, the displacement of the valve from mid-position would be

$$CN = SM + CM \tan \beta = CA \cos \theta + CB \sin \theta \tan \beta \quad (40)$$

the primary motion of the valve is therefore harmonic, the component virtual eccentrics being

$$a = CA = \frac{q_2}{s_2} k \quad \ldots \ldots \ldots \ldots \quad (41)$$

$$b = CB \tan \beta = \frac{q}{s} k \tan \beta = \frac{q}{s} \cdot \frac{k}{l} \cdot h \quad \ldots \ldots \quad (42)$$

from which—

$$\left.\begin{aligned} \tan\alpha\tan\beta &= \frac{s \cdot q_2}{q \cdot s_2} \\ \text{or} \qquad \tan\alpha &= \frac{s \cdot q_2 \cdot l}{q \cdot s_2 \cdot h} \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (43)$$

and the locus of the virtual eccentric centre is again a straight line.

Secondary Displacements.—Each of these is treated separately, as before.

(1) Angular oscillation of S L. In fig. 156, with centre L, draw arc S n; then C n = O L, and the deviation required

$$= \mathrm{N}\,n = \frac{\mathrm{S\,N}^2}{2\,.\,\mathrm{S\,L}}\sec\beta \text{ nearly} = \frac{\mathrm{C\,M}^2 \sec^2\beta}{2\,l}\sec\beta,$$

that is—

$$\delta_1 = \frac{q^2\,k^2 \sec^3\beta}{2\,s^2\,l}\sin^2\theta \quad . \; . \; . \; . \; . \; . \quad (44)$$

(2) Effect due to rod Q_2 R. The deviation of the valve on this account is the same as the lateral deviation of S, which is

$$\frac{\mathrm{S\,P}_2}{\mathrm{Q}_2\,\mathrm{P}_2} \text{ that of } \mathrm{Q}_2 \text{ or } \frac{p_2}{s_2}\,.\,\frac{\mathrm{O\,Q}^2}{2\,r},$$

fig. 154; therefore—

$$\delta_2 = -\,\frac{q^2\,p_2\,k^2}{2\,s^2\,s_2\,r}\sin^2\theta \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (45)$$

(3) and (4) Oscillation of the "spanner" E P_1.

Referring to fig. 154, the horizontal displacements of P_0 and P_1 from mean position are, respectively,

$$k\cos\theta - \frac{p}{s}\,.\,\frac{k}{2\,\mu}\sin^2\theta \text{ and } \frac{s_1}{s_2}\left(k\cos\theta - \frac{k}{2\,\mu}\sin^2\theta\right);$$

the difference of the two,

$$\frac{q_1}{s_2}k\cos\theta - \frac{k}{2\,\mu}\left(\frac{p}{s} - \frac{s_1}{s_2}\right)\sin^2\theta,$$

representing the horizontal motion of P_1 relative to P_0, is approximately harmonic, since the second term in the expression is always proportionately small. This relative horizontal motion disturbs the valve only so far as it causes the vertical displacements of P_1 and P_0 to differ, and this effect may be divided into two parts, viz., (3) that due to the swing of the "spanner," or the motion of P_1 in a circular arc relative to the connecting rod, supposing the latter very long, and therefore to remain parallel to itself;

(4) that due to the angle of the connecting rod, assuming the spanner very long, and the point P_1 consequently to oscillate along the axis of the rod P_0.

Deviations higher than secondary being neglected, (3) and (4) may be calculated separately, the result being that the former (see fig. 155)

$$= G\,g = \frac{D\,P_1{}^2}{2\,E\,P_1}\sin^2\theta = \frac{q^2{}_1\,k^2}{2\,s_2{}^2\,e}\sin^2\theta,$$

$$\text{and the latter} = D\,G\tan\phi = \frac{q_1}{s_2}\,k\cos\theta\sin\phi \text{ nearly,}$$

$$= \frac{q^1}{s_2}\,k\cos\theta\,\frac{\sin\theta}{\mu}$$

ϕ being the angle of the connecting rod.

Multiplying by $\tan\beta$ to obtain the effect on the valve,

$$\delta_3 = -\,\frac{q_1{}^2\,k^2\,\tan\beta}{2\,s_2{}^2\,e}\sin{}^2\,\theta \quad . \; . \; . \; . \; . \; . \quad (46)$$

$$\delta_4 = \frac{q_1\,k\,\tan\beta}{2\,s_2\,\mu}\sin 2\,\theta \quad . \; . \; . \; . \; . \; . \; . \quad (47)$$

(5) Obliquity of P_1 S. The vertical deviation of S due to the swing of this rod

$$= \frac{S\,P_1}{Q_2\,P_1}\,.\,J\,j = \frac{p_1}{s_1}\,\frac{(C_0\,P_1)^2}{2\,Q_2\,P_1}\sin^2\theta,$$

and $$\delta_5 = \frac{p_1\,k^2\,\tan\beta}{2\,s^2{}_2}\sin^2\theta \quad . \; . \; . \; . \; . \; . \; . \quad (48)$$

(6) Effect on the valve of the deviation in the motion of the crosshead. The lateral displacement of S, which $= a \cos\theta$, is derived directly from the motion of the crosshead, and therefore has the same proportionate deviation as the latter, or

$$\delta_6 = -\,\frac{q_2}{s_2}\,\frac{k}{2\,\mu}\sin^2\theta = -\,\frac{a}{2\,\mu}\sin^2\theta \quad . \; . \; . \quad (49)$$

This is therefore exactly equal to x, one component of the required compensation, a property common to other gears, such as the Waldegg-Walschaert, in which the motion a is obtained from the crosshead.

(7) Obliquity of P_2 T. As before, the effect of this is small, and is given by

$$\delta_7 = \frac{q^2\,q_2\,k^2}{2\,s^2\,s_2\,t}\sin^2\theta \quad . \; . \; . \; . \; . \; . \; . \quad (50)$$

(8) Angular oscillation of reversing rod F L. The effect of this is to produce a small sliding of the block L in the sector H; if δh denote the amount of this for any crank position θ, then, comparing equations (40) and (42), it is seen that the corresponding deviation of the valve is

$$\delta b \sin \theta = \frac{q}{s} \cdot \frac{k}{l} \cdot \delta h \sin \theta.$$

But

$$\delta h = (\text{valve displacement})^2 \times \frac{\sec \beta}{2f} = \frac{\{\rho \sin (\alpha + \theta)\}^2}{2f} \sec \beta,$$

whence

$$\delta_8 = \frac{\rho^2 q k \sec \beta}{2 s l f} \sin \theta \sin^2 (\alpha + \theta) \quad . \; . \; . \; . \; . \; . \quad (51)$$

This completes the account of the secondary displacements; collecting and comparing the results in the same manner as before, remembering that $\delta_6 = x$, the following equations are found :—

Average value of $\delta_1 + \delta_2 + \delta_7 = 0$, or

$$\frac{1 + \sec^3 \beta}{2l} + \frac{q_2}{s_2} \cdot \frac{1}{t} = \frac{p_2}{s_2} \cdot \frac{1}{r} \quad . \; . \; . \; . \; . \quad (52)$$

also $\delta_3 + \delta_5 = 0$, or

$$q_1^2 = p_1 e \quad . \; . \; . \; . \; . \; . \; . \; . \quad (53)$$

and $\delta_4 = y$, or

$$\frac{q_1}{s_2} = \frac{1}{2} \frac{q}{s} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (54)$$

Equation (52) is to be used for calculating the length of the radius rod Q_2 R. From equation (53) the length of the spanner E P_1 is determined in order that the effect of its oscillation may counteract that of the rod Q_2 S, and so cause the vertical movement of S to be the same above as below the neutral position, fig. 155. In accomplishing this, a disturbance is introduced owing to the angular oscillation of the connecting rod; but this is taken advantage of, since it is of the nature required to give the compensation y, equation (8), for all settings of gear. To find the proportions for securing the best result, equation (54) is used, which is equivalent to the condition that D P_1, fig. 155, shall be equal to half the greatest distance of P_0 from the centre line C Q.

In order to limit the extent of the swing of the spanner, D P_1 may in some cases require to be somewhat greater than is here indicated.

An examination of equation (51) shows that the unbalanced deviation δ_8 acts symmetrically in the two strokes, and so produces no inequality in the action of the two sides of the valve, even if the reversing rod L F be comparatively short.

From the analysis, it is inferred that the Morton gear is capable of giving an excellent steam distribution for all grades of expansion.

In estimating the degree of accuracy attained in these investigations, and the relative value of the deviations of higher orders neglected in the analysis, it is found that such deviations can be all expressed in the form $A \sin^m \theta \cos^n \theta + B$, where m and n are positive integers whose sum is not less than three, and A is a small quantity depending on the angles of swing of the rods and on the grade of expansion, and is of the order $m + n - 1$, unless one or more of the angular oscillations be excessive. When $m + n$ is odd, the corresponding disturbance acts symmetrically in the two strokes, and the effect on the velocity of the valve need only be considered; a deviation of this nature, which increases the velocity of cut-off without proportionately increasing the travel, must be looked on as an advantage. When $m + n$ is even, the action is not symmetrical, but the effect is minute on account of the smallness of the factor A. The constant B is neutralised by simply altering the length of the valve rod.

In conclusion, the method of analysis illustrated in this article is applicable to link motions and to all gears except those in which there is no approach to harmonic displacement, such as trip gears, and arrangements actuated by cams.

THE WILLANS EXPANSION VALVE GEAR.

The Willans engine is single-acting, and the piston rod is replaced by a hollow trunk pierced with steam passages, fig. 157, in which trunk works a piston valve, which is driven by an eccentric on the crank pin, so that the motion of this valve to the passages is the same as if they were fixed and the eccentric on the shaft. The piston valve forms the distribution valve, while the trunk, which passes through a gland on each cylinder top, forms the cut-off or expansion valve. It will be seen that the whole arrangement is equivalent to an ordinary slide valve and a cut-off valve, which is driven by an eccentric having a throw equal to the crank, but placed at 180 deg. to it, this cut-off valve working over fixed passages, and not passages in the distribution valve. But in Willans' arrangement the clearance is very small

compared to what it would be in this. Cut-off can be varied by having the cut-off passages cut in a helix, fig. 158, round which a helical sleeve works. The sleeve is rotated, which is equivalent to an increase of outside lap in the expansion valve. The cut-off can also be permanently altered by altering the height of the gland in throttling engines.

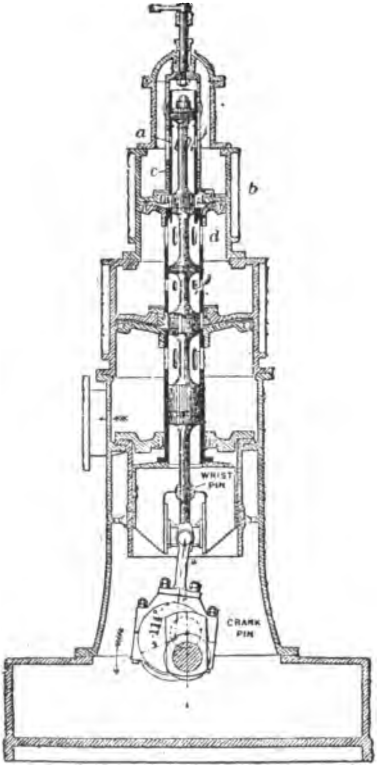


FIG. 157.

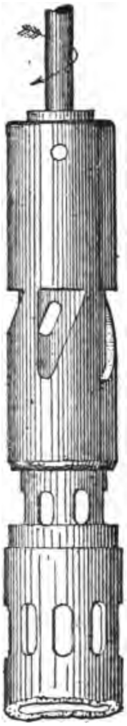


FIG. 158.

CHAPTER XVI.

ON THE DISTURBANCES CAUSED BY THE MOVING PARTS OF ENGINES, AND THE METHODS OF BALANCING THEM.

WHEN an engine is at rest, whatever force may act on the piston, the forces on the engine as a whole are in equilibrium; but when it is in motion some of these forces are causing acceleration or retardation of the moving parts, so that they are not in equilibrium, and these forces, continually changing in magnitude and direction, produce vibrations which in some cases it is necessary to reduce as much as possible.

We shall first consider (fig. 159) the case of an engine having two cranks at right angles, the centres of whose cylinders are $2c$ ft. apart, and whose stroke is $2p$ ft. Let M be the weight of the crank pin and of one of the webs; let ω be the angular velocity of the crank. The centrifugal forces produce a resultant—

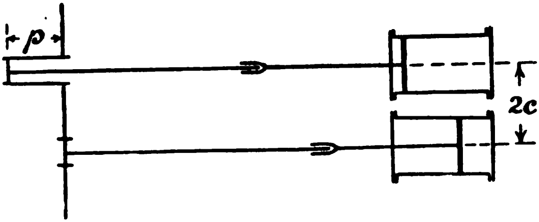


FIG. 159.

$$F_1 = \frac{M}{g} p \omega^2 \sqrt{2},$$

acting through the centre of the shaft midway between the cranks, and inclined at 45 deg. to each crank. It also produces a couple—

$$\frac{M}{g} p \omega^2 c \sqrt{2}.$$

For let F, F be the centrifugal forces, fig. 161; these may be resolved into forces at right angles at 45 deg. to F, F, namely, $\frac{F_1}{2}$, $\frac{F_1}{2}$, and G, G.

$$\frac{F_1}{2} = G = \frac{M p \omega^2}{g} \frac{\sqrt{2}}{2}$$

The forces G, G form the couple, and $\frac{F_1}{2}$, $\frac{F_1}{2}$ the resultant force F_1.

As the acceleration and retardation of the connecting rod is difficult to calculate, it is usual to divide its weight into two parts. The large end and half its length are supposed to be collected at the crank pin; and the remainder at the crosshead. Let, now, M represent the weight of crank pin, one of the webs, the large end of the connecting rod and one half its length, instead of the first two alone. The reciprocation of the piston rod, and, according to our supposition, a part of the weight of the connecting rod, subtracts from the effective pressure of the steam on each piston a force

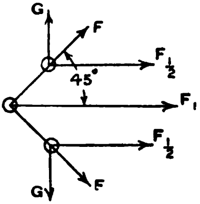


FIG. 161.

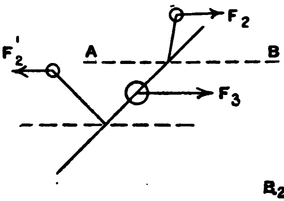


FIG. 162.

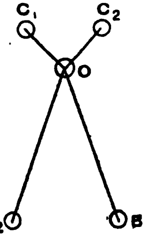


FIG. 163.

$$F_2 = \left\{ \cos\theta + \frac{n^2(\cos^2\theta - \sin^2\theta) + \sin^4\theta}{(n^2 - \sin^2\theta)^{\frac{3}{2}}} \right\} \frac{M_1}{g} p\,\omega^2$$

Where M_1 is the weight of the reciprocating parts, θ is the angle made by the crank with the line of stroke, and n the ratio of the length of connecting rod to crank, so that with a very long connecting rod

$$F_2 = \frac{M_1}{g} p\,\omega^2 \cos\theta.$$

If θ be the angle made by the following crank with the line of stroke A B, fig. 162, then the two forces $F_2, F_2{}^1$ produced by the reciprocating parts are equivalent to a resultant

$$F_3 = \frac{M_1}{g} p \omega^2 (\cos\theta - \sin\theta),$$

and a couple in the direction shown by O in fig. 162, which is equal to

$$T = \frac{M_1}{g} c\, p\, \omega^2 (\cos\theta + \sin\theta).$$

F_3 is the greatest when $\theta = -45$ deg., when it equals

$$F_4 = \frac{M_1}{g} p \omega^2 \sqrt{2},$$

and T is greatest when $\theta = 45$ deg., when its value is

$$T_1 = \frac{M_1}{g} c\,.\,p\,.\,\omega^2 \sqrt{2}.$$

In locomotives it is usual to partially balance by means of weights placed on the wheels, and we shall first show how to find the position and magnitude of these balance weights. These are so arranged that their centrifugal force neutralises the couple T when $\theta = 45$, and also F_3 when $\theta = 45$ deg., but when balancing the couple there is an unbalanced vertical force, and when balancing the force there is an unbalanced vertical couple. They must therefore produce a resultant force—

$$R = \frac{(M + M_1)}{g} p\, \omega^2 \sqrt{2}$$

and a couple

$$T^2 = \frac{M + M_1}{g} c\, p\, \omega^2 \sqrt{2}.$$

In fig. 163 let C_1, C_2 represent the two crank pins of a locomotive having inside cylinders, and B_1, B_2 the balance weights on the wheels nearest to each of the above cranks, and let 2ψ be the angle B_1, O B_2; then

$$R = \frac{2m}{g} r\,.\,\omega^2\,.\,\cos\psi$$

$$T_2 .= \frac{2m}{g} a\,.\,r\,\omega^2 \sin\psi$$

where $2a$ is the horizontal distance between the centres of gravity of the balance weights, and m is the mass of each, and r its radius of revolution.

$$\therefore \frac{T_2}{R} = a \tan \psi = c$$

$$\tan \psi = \frac{c}{a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (A)$$

$$\sqrt{R^2 + \frac{T^2}{a^2}} = \frac{2m}{g} r \omega^2 = \frac{M + M_1}{g} p \omega^2 \sqrt{2} \sqrt{\frac{a^2 + c^2}{a^2}}$$

$$m \,.\, r \,. = (M + M_1) p \sqrt{\frac{a^2 + c^2}{2a^2}} \quad . \quad . \quad . \quad . \quad (B)$$

Equations (A) and (B) give the magnitude and position of the balance weights where there are no coupling rods. It is not always the practice to balance the reciprocating parts; in the United Kingdom the fraction balanced is sometimes as small as one-third. This will not alter ψ, but we must multiply M_1 in equation (B) by this fraction.

Another and simpler method of finding the balance weights is the following. It is clear from the above that the balance would be perfect if the weights of the reciprocating parts revolved with their respective crank pins. We begin, therefore, by assuming that two weights, each equal to $M + M_1$, are at E and F, fig. 164. Let G and K be two weights on the two wheels, opposite to the crank F at radii p, whose centrifugal force would balance $M + M_1$ at F. Then

$$2a \,.\, K \,. = (M + M_1)(c + a).$$

$$2a \,.\, G = (M + M_1)(a - c).$$

$$K = H = \frac{(M + M_1)(c + a)}{2a.}$$

$$G = L = \frac{(M + M_1)(a - c)}{2a}$$

$$\therefore \sqrt{K^2 + L^2} = (M + M_1) \sqrt{\frac{(c^2 + a^2)}{2a^2}} = \frac{m \,.\, r}{p}$$

which gives the magnitude of the single weight m at radius r, which is equivalent to K and L at radius p; also if θ be the angle made by the radius from B to m with B K, then

$$\tan \theta = \frac{L}{K} = \frac{a - c}{a + c}.$$

If the cylinders are outside the wheels, let M_2 be the weight of each crank, M the weight of the part of the pin outside the crank plus the big end of the connecting rod and half its length, while M_1 is the weight of the reciprocating parts plus the small end of the connecting rod and half its length, or, if only a part of these are to be balanced, let M_1 represent that fraction.

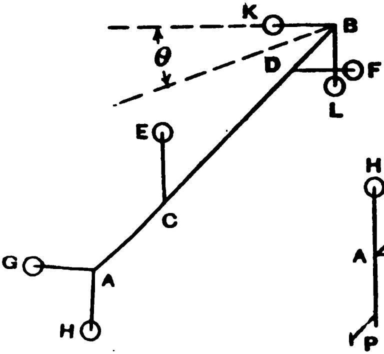


Fig. 164.

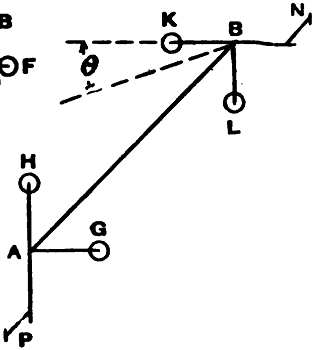


Fig. 165.

Fig. 165 shows the two cranks at right angles. K and G balance the further crank and the parts connected with it, H and L the nearer. All are supposed at first to be at radius p; then, taking moments about B,

$$2aG = (M + M_1)(c - a),$$

and, taking moments about A,

$$2a(K - M_2) = (M + M_1)(c + a),$$

from which we can find G and K, which equal L and H respectively.

$$\frac{mr}{p} = \sqrt{K^2 + L^2},$$

as before, and

$$\tan\theta = \frac{L}{K}.$$

But, whereas with inside cylinders a side elevation would show the angle between the radii to the centres of gravity of the balance weights to be less than two right angles by 2θ, with outside cylinders it is greater than two right angles by this amount.

A numerical example of this last case is as follows: Let

$$M + M_1 = 423 \text{ lb.},$$
$$M_2 = 117 \text{ lb.},$$
$$2a = 55 \text{ in., and } 2c = 74 \text{ in.}$$

Then
$$(K - M_2) = \frac{a + c}{2a}(M + M_1)$$
$$= \frac{74 + 55}{2 \times 55} \times 423 = 496 \text{ lb.}$$
$$K = 496 + 117 = 613 \text{ lb.}$$
$$G = (M + M_1)(c - a) = \frac{74 - 55}{2 \times 55} \times 423 = 73 \text{ lb.}$$
$$\frac{mr}{p} = \sqrt{(613)^2 + (73)^2} = 617 \text{ lb.}$$
$$\tan\theta = \frac{G}{K} = \frac{73}{613} = \cdot 119.$$

If there are coupling rods with outside cylinders, and $2l$ is the horizontal distance between the length of the rods, then, when there are four wheels coupled, half the weight of each rod must be balanced on each wheel; if six wheels, then on each of the leading and trailing wheels one-half the weight of each rod, and on each driving wheel the whole weight of one rod, must be balanced. This is evident, because the weight of each rod may be equally divided between its two ends. The same applies to inside cylinders, but the cranks on the wheels must also be taken into account.

Another method of balancing is by reciprocating weights, which has been applied to locomotives, but has been given up, being found inferior to the above plan. It is, however, used in marine engines by Mr. Yarrow, and reduces the vibration considerably. The revolving parts are first balanced by counter-weights on the cranks and the reciprocating parts by means of two "bob weights," as shown in fig. 166. We have here taken the simplest possible case of a piston &c., with a stroke of 2 ft., the weight of the reciprocating parts being 300 lb. Two eccentrics are placed on the shaft opposite to the crank, one at 2 ft. and the other at 4 ft. from the line of stroke, the stroke of each eccentric being 6 in. It is clear, then, that the sum of the bob weights must be four times that of the reciprocating weights, because the eccentric stroke is one-fourth that of the piston. Again, by the principle of the lever, the weights must be 400 lb. and 800 lb.

Fig. 166A shows the three cranks and pairs of eccentrics of a triple-expansion marine engine. The reciprocating bob weights are to be driven by eccentrics on the lines X, Y, these being the most suitable positions, and the motions of the valves are to be considered when they are in full forward gear. Each unbalanced moving part is now dealt with separately, as before described, and the position and amount of the weights necessary to balance it ascertained, the stroke of the balance weights being taken for purposes of calculation as equal to the stroke of the part they balance in each case. For instance, the unbalanced reciprocating parts of the intermediate pressure piston, piston rods, &c., lettered B, fig. 166A, weigh 162 lb.; the balance required at X is found to be 81·8 lb., and that at Y 80·2 lb., the stroke of each being 16 in. Taking another instance, for example, the high-pressure valve and its go-ahead gear lettered E weighing

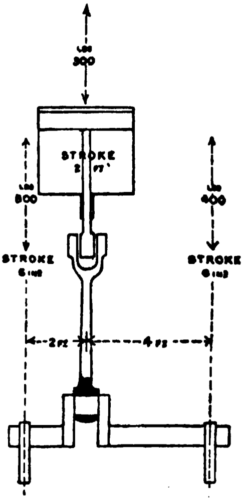


FIG. 166.

TRAVEL OF PISTONS 16 ins
— — VALVES 5 ins

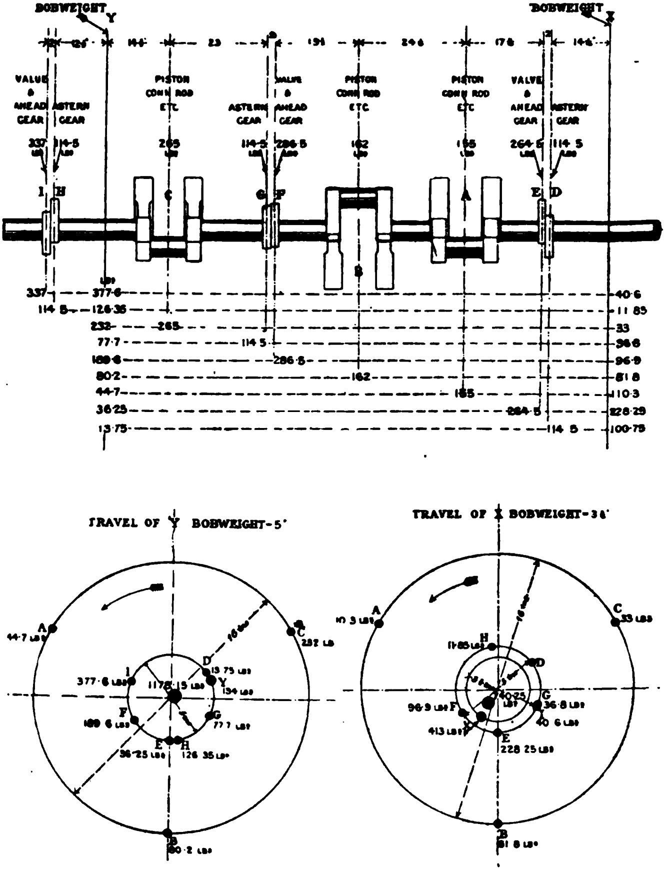


FIG. 166A.

264·5 lb., the balance at X is 228·25 lb., and at Y 36·25 lb., the stroke of each being 5 in. After dealing in a similar manner with all the reciprocating parts, if we were to construct two uniform discs with each of the weights thus found pinned on in its proper relative position, and place them respectively at X at Y, the engine would be balanced vertically. All the weights at X may be replaced by one large one, equal to their sum, and having the position of their centre of gravity. The weights at Y may be dealt with in like manner. These are shown by the large black spots on the diagrams to the right and left of the middle figure. These again might be substituted by larger or smaller weights as convenient, the product of weight and stroke being kept constant. In the present case the total weight of all the balances is 740·25 lb. at X., and 1,178·15 lb. at Y, and their strokes are 2·08 in. and ·48 in. respectively. These would be equivalent to 413 lb. at X with a stroke of 3¾ in., and 134 lb. at Y with a stroke of 5 in. If these weights rotate, they will only balance vertically, but if they reciprocate they will balance completely, neglecting the effect of the obliquity of connecting rods.

SCHLICK'S SYSTEM OF BALANCING THE RECIPROCATING PARTS OF ENGINES.

This method requires four cylinders and four cranks. The two that have the heavier moving parts are placed in the middle. The reciprocating forces produce a resultant force and a resultant couple, both of which must be balanced. In this method, however, the obliquity of connecting and eccentric rods is not considered. Let the cylinders be numbered from the left I., III., IV., II.; let P_3 and P_4, the weights of the reciprocating parts of the central cylinders, be known.

First Case.—Let the angle between their cranks be 90 deg., and suppose that all the cranks have an equal throw, and that the distance from centre to centre of cylinders is the same; then we have to find the weights P_1, P_2 of the cylinders I., II., and the positions of their cranks. In order to balance the reciprocating parts of cylinder IV. alone, we might have weights placed on the centre lines of I. and II. opposite to the crank of cylinder IV., the weights being $\frac{1}{3}P_4$ for cylinder I., and $\frac{2}{3}P_4$ for cylinder II. In the same way, to balance P_3 alone, we should require two weights, $\frac{2}{3}P_3$ and $\frac{1}{3}P_3$ on the centre lines of P_1 and P_2, and opposite to P_3's crank. We can, by finding the resultant of these loads both in direction and magnitude, assign directions to the cranks of I. and II., so that their reciprocating parts will balance those of III. and IV. Let us consider crank II., in fig. 167. We see that if P_2's crank makes an angle α with the direction of P_4 produced, and $90 - \alpha$ with the direction of P_3 produced, then

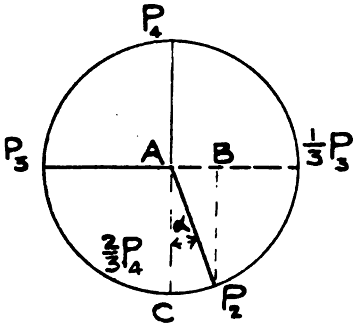


FIG. 167.

$$P_2 \cos \alpha = \tfrac{2}{3} P_4.$$
$$P_2 \sin \alpha = \tfrac{1}{3} P_3.$$
$$P_2{}^2 = (\tfrac{2}{3} P_4)^2 + (\tfrac{1}{3} P_3)^2.$$

Hence
$$\tan \alpha = \frac{P_3}{2 P_4}$$

Secondly, if β is the angle the crank of P_1 makes with P_3 produced at 90 deg. $- \beta$, the angle it makes with P_4 produced, then

$$P_1{}^2 = (\tfrac{2}{3} P_3)^2 + (\tfrac{1}{3} P_4)^2$$

and
$$\tan \beta = \frac{P_4}{2 P_3}.$$

The arrangement is shown in fig. 168.

Second Case.—Let the angle between the cranks III. and IV. be γ, not a right angle. Consider P_1, and let it make an angle β with P_3 produced, fig. 169. Then, reasoning as before, P_1 may be divided into two parts equal to $\frac{2}{3} P_3$, and $\frac{1}{3} P_4$ set on the transverse plane passing through P_1's centre line, the former opposite to P_3's crank, and the latter opposite to P_4's crank. Hence

$$P_1{}^2 = (\tfrac{2}{3} P_3)^2 + (\tfrac{1}{3} P_4)^2 + \tfrac{4}{9} P_3 P_4 \cos \gamma.$$

$$P_2{}^2 = (\tfrac{2}{3} P_4)^2 + (\tfrac{1}{3} P_3)^2 + \tfrac{4}{9} P_3 P_4 \cos \gamma.$$

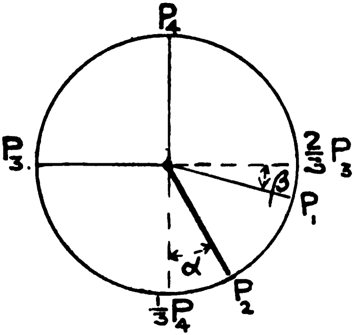


FIG. 168.

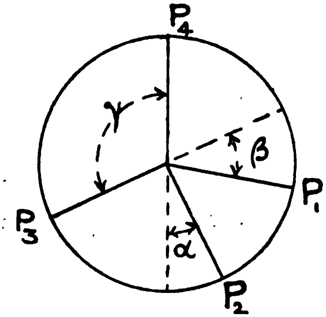


FIG. 169.

Also
$$\sin \alpha = \tfrac{1}{3} \frac{P_3}{P_2} \sin \gamma.$$

$$\sin \beta = \tfrac{1}{3} \cdot \frac{P_4}{P_1} \sin \gamma.$$

Third Case.—The weights acting upon the outer cranks and the angles formed by the said cranks may be known, and from this we can obtain the angular position of the cranks, as well as the weights that should act upon two intermediate cranks, and if the angle δ is a right angle, we have—

$$P_4{}^2 = 2P_2{}^2 + P_1{}^2$$

$$P_3{}^2 = 2P_1{}^2 + P_2{}^2$$

$$\tan \epsilon = \frac{P_1}{2P_2};$$

$$\tan \phi = \frac{P_2}{2P_1}.$$

If the angle δ is not a right angle, the formulæ will be (fig. 170)

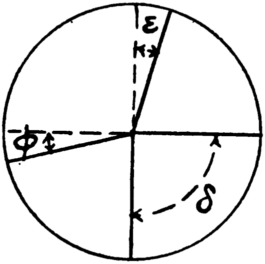


FIG. 170.

$$P_4{}^2 = 2P_2{}^2 + P_1{}^2 - 4P_1P_2 \cos \delta$$

$$P_3{}^2 = 2P_1{}^2 + P_2{}^2 - 4P_1P_2 \cos \delta$$

and $$\sin \epsilon = \sin \delta \frac{P_1}{P_4}$$

$$\sin \phi = \sin \delta . \frac{P_2}{P_3}.$$

In determining the weights that act upon the various cranks the revolving elements must also be considered, besides the reciprocating elements, but, of course, these must be balanced by weights each opposite its own crank, and it must be remembered that revolving forces cannot balance reciprocating forces perfectly. The effect of the valve gear or other moving elements may be taken account of in two ways (no allowance being made for obliquity). If the centres of slide valves lie in the same plane as the piston rods, the weights of moving elements of valve gearing can be taken account of when angles of cranks, &c., are calculated, and forces exerted by valve gearing balanced simultaneously with forces exerted upon the cranks. If, however, the slide valves or rods be in a different plane from that of the piston rods, it will only be necessary to so arrange that the distances between the valve rods, the weights of the moving parts of the valve gear, the radii and angles of the eccentrics, shall have the same relation to one another as the moving elements of cylinders and cranks.

If, for instance, the distance between the valve rods is equal to the distances between the cylinder centres, and if all the eccentrics have an equal throw and an equal angle of advance, it is only necessary to determine the weights of the moving elements of the valve gear in the same manner as that resorted to for the determination of the cylinder weights. Should some of the valves differ, it will in most cases still be possible to balance the forces of their moving elements by choosing the other or remaining factors accordingly. In order, however, to perfectly balance the said forces it is necessary that the centres of gravity be situated in the centres of the slide rods, and that the latter lie in one and the same plane.

Balancing when the effect of the connecting rod's obliquity is included.

The formula for the accelerating force may be expressed with sufficient accuracy as

$$F = \frac{W r \omega^2}{g}\left(\cos\theta + \frac{\cos 2\theta}{c}\right)$$

where $c\,r$ is the length of the connecting rod, θ the angle made by the crank with the line of stroke, and ω the angular velocity.

Let an engine have n cranks, and let $a_1, a_2, a_3, \ldots a_{n-1}$ be the angles between the second and first, third and first, and so on. Let $l_1, l_2, l_3, l_4 \ldots l_{n-1}$ be the distances between the centre lines of the cylinders, and W_o, $W_1 \ldots W_{n-1}$ be the weights of the reciprocating parts. Then, supposing the rotating parts are balanced, the sum of the vertical forces (the engine being vertical) produced at any instant must be zero, and their moments about any axis, say an axis perpendicular to the shaft and passing through the line of stroke of the first piston, must be zero. This is expressed mathematically by the two equations :

$$W_0\left(\cos\theta+\frac{\cos 2\theta}{c}\right)+W_1\left\{\cos(\theta+\alpha_1)+\frac{\cos 2(\theta+\alpha_1)}{c}\right\}$$
$$+\ \&c. = 0\,;$$

and $$W_1 l_1\left\{\cos(\theta+\alpha_1)+\frac{\cos(2\theta+2\alpha_1)}{c}\right\}+\&c. = 0.$$

If $\theta = 0$,

$$W_0\left(1+\frac{1}{c}\right)+W_1\left(\cos\alpha_1+\frac{\cos 2\alpha_1}{c}\right)+\&c. = 0\,;$$

and if $\theta = 180$,

$$W_0\left(\frac{1}{c}-1\right)+W_1\left(\frac{\cos 2\alpha_1}{c}-\cos\alpha_1\right)+\&c. = 0.$$

Hence

$$W_0+W_1\cos\alpha_1+W_2\cos\alpha_2+\&c. = 0\,;$$

and

$$W_0+W_1\cos 2\alpha_1+W_2\cos 2\alpha_2+\&c. = 0.$$

Again, if $\theta = \frac{\pi}{2}$

$$-\frac{W_0}{c}+W_1\left(-\frac{\cos 2\alpha_1}{c}-\sin\alpha_1\right)$$

$$\frac{W_0}{c}+W_1\left(\sin\alpha_1+\frac{\cos 2\alpha_1}{c}\right)+\&c. = 0.$$

Hence $$W_1\sin\alpha_1+W_2\sin\alpha_2+\&c. = 0,$$

and by expanding

$$W_0\left(\cos\theta+\frac{\cos 2\theta}{c}\right)+W_1\left(\cos\theta\cos\alpha_1-\sin\theta\sin\alpha_1\right)$$
$$+\frac{W_1}{c}\left(\cos 2\theta\cos 2\alpha_1-\sin 2\theta\sin 2\alpha_1\right)+\&c. = 0.$$

But we have already shown that the coefficients of $\cos\theta$, $\cos 2\theta$, and $\sin\theta$ are zero; hence the coefficient of $\sin 2\theta$ must also be zero;

$$\therefore\ W_1\sin 2\alpha_1+W_2\sin 2\alpha_2+\&c. = 0.$$

Treating the second equation of moments in the same way,

$$W_1 l_1\cos\alpha_1+W_2 l_2\cos\alpha_2+\&c. = 0.$$
$$W_1 l_1\sin\alpha_1+W_2 l_2\sin\alpha_2+\&c. = 0.$$
$$W_1 l_1\cos 2\alpha_1+W_2 l_2\cos 2\alpha_2+\&c. = 0.$$
$$W_1 l_1\sin 2\alpha_1+W_2 l_2\sin 2\alpha_2+\&c. = 0.$$

Hence we have eight equations, and the unknown quantities are

$$\frac{W_1}{W_0}, \frac{W_2}{W_0}, \frac{W_3}{W_0}, \text{\&c.}; \quad \alpha_1, \alpha_2, \alpha_3, \text{\&c.}; \quad \frac{l_2}{l_1}, \frac{l_3}{l_1}, \text{\&c.}$$

Let us first consider the case of an engine with three cranks. Here

$$W_0 + W_1 \cos \alpha_1 + W_2 \cos \alpha_2 = 0.$$
$$W_1 \sin \alpha_1 + W_2 \sin \alpha_2 = 0.$$
$$W_0 + W_1 \cos 2\alpha_1 + W_2 \cos 2\alpha_2 = 0.$$
$$W_1 \sin 2\alpha_1 + W_2 \sin 2\alpha_2 = 0.$$
$$W_1 l_1 \cos \alpha_1 + W_2 l_2 \cos \alpha_2 = 0.$$
$$W_1 l_1 \sin \alpha_1 + W_2 l_2 \sin \alpha_2 = 0.$$
$$W_1 l_1 \cos 2\alpha_1 + W_2 l_2 \cos 2\alpha_2 = 0.$$
$$W_1 l_1 \sin 2\alpha_1 + W_2 l_2 \sin 2\alpha_2 = 0.$$

It follows from the second and sixth of these that $l_1 = l_2$ a manifest impossibility. Hence engines with three cranks cannot be self-balancing, but require the addition of other cranks or eccentrics, as in Yarrow's method of balancing.

If we have four cranks,

$$W_1 \sin \alpha_1 + W_2 \sin \alpha_2 + W_3 \sin \alpha_3 = 0,$$
$$W_1 l_1 \sin \alpha_1 + W_2 l_2 \sin \alpha_2 + W_3 l_3 \sin \alpha_3 = 0;$$

hence $\quad W_2 (l_1 - l_2) \sin \alpha_2 = W_3 (l_3 - l_1) \sin \alpha_3,$

or
$$\frac{W_2 \sin \alpha_2}{l_3 - l_1} = \frac{W_3 \sin \alpha_3}{l_1 - l_2}$$
$$= \frac{W_1 \sin \alpha_1}{l_2 - l_3}.$$

Again, $l_1 W_1 \sin 2\alpha_1 + l_2 W_2 \sin 2\alpha_2 + l_3 W_3 \sin 2\alpha_3 = 0,$

$$W_1 \sin 2\alpha_1 + W_2 \sin 2\alpha_2 + W_3 \sin 2\alpha_3 = 0;$$

$$\therefore \frac{W_2 \sin 2\alpha_2}{l_3 - l_1} = \frac{W_3 \sin 2\alpha_3}{l_1 - l_2} = \frac{W_1 \sin 2\alpha_1}{l_2 - l_3}.$$

Then either $\alpha_1, \alpha_2, \alpha_3$ are each $= 0$ or π, or $\cos \alpha_2 = \cos \alpha_3 = \cos \alpha_1$.

In the latter case two at least of $\alpha_1, \alpha_2, \alpha_3$ must be equal to one another. These will be α_3 and α_1, because

$$\frac{W_3}{l_1 - l_2} \text{ and } \frac{W_1}{l_2 - l_3}$$

are both negative, and $\alpha_2 = 2\pi - \alpha_1$, because $\dfrac{\sin \alpha_2}{l_3 - l_1}$ is of the same sign as $\frac{\sin \alpha_1}{l_2 - l_3}$, and ∴ sin α_2 must be of the opposite sign to sin α_1; hence

$$\frac{W_2}{l_1 - l_3} = \frac{W_3}{l_1 - l_2} = \frac{W_1}{l_2 - l_3}.$$

But $l_1 W_1 \cos \alpha_1 + l_2 W_2 \cos \alpha_2 + l_3 W_3 \cos \alpha_3 = 0.$

Hence, dividing by cos α_1 and substituting for W_1, W_2, W_3, we have

$$l_1 (l_2 - l_3) + l_2 (l_1 - l_3) + l_3 (l_1 - l_2) = 0,$$
$$2 l_1 l_2 - 2 l_2 l_3 = 0.$$

∴ $l_1 = l_3$, which is absurd.

Considering the case when

$$\sin 2 \alpha_1 = \sin 2 \alpha_2 = \sin 2 \alpha_3 = 0,$$
$$\sin \alpha_1 = \sin \alpha_2 = \sin \alpha_3 = 0,$$

α_1, α_2, α_3 must equal 0 or π. In any case,

$$W_0 + W_1 \cos 2 \alpha_1 + W_2 \cos 2 \alpha_2 + W_3 \cos 2 \alpha_3 = 0.$$
$$\therefore W_0 + W_1 + W_2 + W_3 = 0;$$

but four positive quantities cannot = 0, so that this is inadmissible, so that α_1, α_2, α_3 cannot produce equilibrium if they are 0 or π.

Hence, even with four cranks, an engine cannot be properly balanced without the addition of an auxiliary crank or eccentric carrying balance weights.

If, however, an engine has five or six cranks; for example, if two triple engines work on the same shaft, but are right and left handed, and the cranks are arranged as follows: the two centre are vertical, the two nearest to these on the right and left are at 120 deg. to the vertical, but in front of the vertical plane through the shaft, while the two furthest are at 120 deg. to the vertical and behind this vertical plane, then, if the moving parts of the six cylinders are all exactly the same, the engine will be perfectly balanced. For, firstly, as regards couples, whatever right-hand couples are produced by one engine, equal and opposite left-hand couples are produced by the other, in whatever position the shaft may be; and the equations that deal with forces are, for each three cranks—

$$1 + \cos 120 + \cos 240 = 0$$
$$\sin 120 + \sin 240 = 0$$
$$1 + \cos 240 + \cos 480 = 0$$
$$\sin 240 + \sin 480 = 0,$$

and these are all identities, so that the vertical forces are all balanced.

If five cranks are used, the centre crank must take the place of the two central cranks of the two triples, and the weight of its piston must be twice that of each of the others.

We shall now discuss a method suggested for the complete self-balancing of four-crank engines, and show wherein the error lies. In fig. 171 let there be two cranks, as shown.

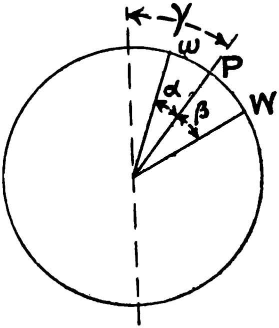


FIG. 171.

Let them make an angle δ, and be acted on by weights w, W. Then, as far as the finite length of the connecting rod is concerned, these have the same effect as a single crank to which the weight P is attached, and whose position between the cranks is fixed by the following equations :

$$w \sin 2\alpha = W \sin 2\beta \quad . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

$$P = w \cos 2\alpha + W \cos 2\beta \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

For this will be attained if

$$P \cos 2\gamma = w \cos 2(\gamma - \alpha) + W \cos 2(\gamma + \beta).$$

(It must be clearly understood that what we have said above only refers to forces, and not to couples, for we can readily see that by properly adjusting γ we can produce a couple from w and W's motion ; *i.e.*, if $\cos 2\gamma$ is zero,

$$w \cos 2(\gamma - \alpha) = - W \cos 2(\gamma + \beta),$$

and the couple cannot be balanced by any single force due to P. Hence, if the couple is to be balanced, it must be by other agency.)

$P \cos 2\gamma = w \cos 2\gamma \cos 2\alpha - w \sin 2\gamma \sin 2\alpha + W \cos 2\gamma \cos 2\beta - W \sin 2\gamma \sin 2\beta$, and if (1) holds good,

$$P \cos 2\gamma = (w \cos 2\alpha + W \cos 2\beta) \cos 2\gamma,$$

which is equation (2).

In the case in which the angle formed by the two cranks is an obtuse angle, they have the same effect as far as the finite length of connecting rod is concerned as a single crank arranged at right angles with a line Op, fig. 172, between the cranks, if the following conditions hold good:

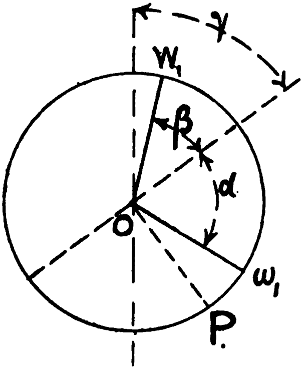


FIG. 172.

$$w_1 \sin 2\alpha_1 = W_1 \sin 2\beta_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and $$P_1 = W \cos(\pi - 2\beta_1) + w_1 \cos(\pi - 2\alpha_1) \quad . \quad . \quad (4)$$

For if so,

$$P_1 \cos(2\gamma + 90 \text{ deg.}) = W_1 \cos(2\gamma - 2\beta_1) + w \cos(2\gamma + 2\alpha_1)$$

$$\therefore -P_1 \cos 2\gamma = W_1 \cos 2\gamma \cos 2\beta_1 + W_1 \sin 2\gamma \sin 2\beta_1 + w_1 \cos 2\gamma \cos 2\alpha_1 - w_1 \sin 2\gamma \sin 2\alpha_1;$$

but applying (3),

$$-P_1 = W_1 \cos 2\beta_1 + w_1 \cos 2\alpha_1,$$

whence (3) follows at once. (Notice the couple is not neutralised.)

Now, if P and P_1 are equal, and these equivalent cranks are placed at right angles, so that the lines O p and O P are in a straight line, as shown in fig. 173, then the forces produced by the reciprocating parts of these cranks, as far as they depend on the obliquity of the connecting rods, will disappear if $P = P_1$, or

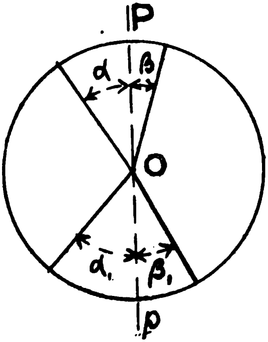


FIG. 173.

$$w \cos 2\alpha + W \cos 2\beta = w_1 \cos(\pi - 2\alpha_1) + W_1 \cos(\pi - 2\beta_1) . \quad (5)$$

and $$w \sin 2\alpha = W \sin 2\beta \quad . \ . \ . \ . \ . \ . \ . \ . \quad (6)$$

$$w_1 \sin 2\alpha_1 = W_1 \sin 2\beta_1 \quad . \ . \ . \ . \ . \ . \quad (7)$$

but it must be noticed that the couples are not necessarily balanced, and there is no doubt that if the distances apart of the cranks were such that these couples would be balanced, that the forces and couples due to that part of the reciprocating forces independent of the obliquity of the connecting rod would not be; in short, the above method, which we believe has been introduced in practice, is merely a laborious attempt to overcome a difficulty that is insurmountable, as we have already shown.

CHAPTER XVII.

ON GOVERNORS.

IF a governor could be constructed without friction, a diagram illustrating its motion could be made such as fig. 174, in which $y_2\,y_1$ are the highest and lowest positions of the sleeve, and the abscissæ of the curve $u_1\,u_2$ represent the angular velocities of the balls, so that when the sleeve is at y the angular velocity of the governor balls is u. But in a real govornor there always exists a resistance R from internal friction, and the diagram that connects the height of sleeve with velocity of rotation is no longer a single line. Suppose, for instance, that the sleeve of the governor under consideration is at the height y, and in equilibrium under the action of the weights of the moving parts, and of the centrifugal force due to the angular velocity u; then, in order to raise the sleeve, the angular velocity must be increased until it reaches some value w, because R must be overcome, and to lower the sleeve the angular velocity must decrease to some value v for the same reason, R being reversed so that it may be represented by − R. Let ω be the mean speed of the governor, then we shall call the "coefficient of sensibility"

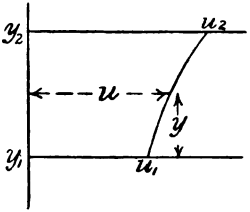


FIG. 174.

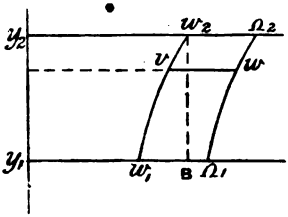


FIG. 175.

$$n = \frac{\omega}{w - v}.$$

The diagram of the governor is therefore a zone, fig. 175, enclosed between two curves, $\Omega_1\,\Omega_2$ and $\omega_1\,\omega_2$, the one the w line, and the other the v line. The governor is thus capable of confining the speed of the engine within the limits ω_1 and Ω_2, and the fraction

$$\frac{\omega}{\Omega_2 - \omega_1} = \mathrm{K}$$

may be termed the coefficient of regularity of the engine.

A governor is termed *astatic* when

$$\omega = \Omega_1 = \omega_2$$

as in fig. 176, so that the load may vary between zero and a maximum while running at its mean speed ω, but is prevented by the governor from passing the limits ω_1, Ω_2.

The term *isochronous* has been applied to a governor for which u is constant, so that in the diagram $u_1\ u_2$ is a vertical straight line. When, however, the resistance R is taken into account, many governors which have been designed with this object, neglecting R, cease to be isochronous.

THE "BUSS" GOVERNOR.

We shall first consider the Buss governor, which is shown in fig. 177; the arm C A B is bent through an angle γ. The point C moves perpendicularly to the axis upon the slide E D and the point A, which carries the sleeve, parallel to the axis G G. The reaction at C is vertical, and obviously equals

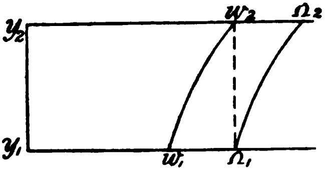


Fig. 176.

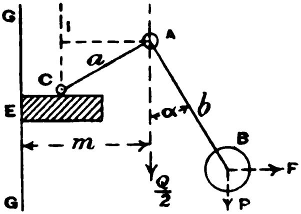


Fig. 177.

$$P + \frac{Q}{2}$$

when the angular velocity is u, and there is no resistance R, P being the weight of one ball and Q that of the sleeve.

The forces on the arm C A B keep it at rest, and we may therefore take moments about A, so that

$$F\,b\cos\alpha = \left(P + \frac{Q}{2}\right) a \sin(\gamma - \alpha) + P\,b\sin\alpha.$$

$$\therefore \frac{u^2 P}{g}(m + b\sin\alpha)\,b\cos\alpha = \left(P + \frac{Q}{2}\right) a\sin\gamma\cos\alpha$$

$$+ \sin\alpha\left\{P\,b - \left(P + \frac{Q}{2}\right) a\cos\gamma\right\} \quad . \quad . \quad . \quad \text{(A)}$$

As the governor is intended to be isochronous, u must remain the same when α is zero. Hence

$$\frac{u^2 P}{g} m b = \left(P + \frac{Q}{2}\right) a \sin \gamma.$$

Substituting this in (A),

$$\frac{u^2 P}{g} m b \cos \alpha = \left(P + \frac{Q}{2}\right) a \sin \gamma \cos \alpha$$
$$+ \sin \alpha \left[P b - \left(P + \frac{Q}{2}\right) a \left\{\cos \gamma + \frac{b \sin \gamma}{m}\right\}\right]$$

If $$P b - \left(P + \frac{Q}{2}\right) a \left\{\cos \gamma + \frac{b \sin \gamma}{m}\right\} = 0,$$

$\cos \alpha$ will divide out, and

$$\frac{u^2 P}{g} m b = \left(P + \frac{Q}{2}\right) a \sin \gamma.$$

Next suppose that u changes to w, so that R is positive.

$$\frac{w^2 P}{g} (m + b \sin \alpha) b \cos \alpha = \left(P + \frac{Q + R}{2}\right) a \sin \gamma \cos \alpha$$
$$+ \sin \alpha \left\{P b - \left(P + \frac{Q + R}{2}\right) a \cos \gamma\right\}. \quad \text{(B)}$$

which is obtained by changing Q to Q + R in equation (A). Then, if possible, let w be constant, so that when $\alpha = 0$

$$\frac{w^2}{g} P m b = \left(P + \frac{Q + R}{2}\right) a \sin \gamma.$$

Substituting this in (B),

$$\frac{w^2 P}{g} m b \cos \alpha = \left(P + \frac{Q}{2}\right) a \sin \gamma \cos \alpha$$
$$+ \frac{R}{2}\left(a \sin \gamma \cos \alpha - a \left\{\cos \gamma + \frac{b \sin \gamma}{m}\right\} \sin \alpha\right)$$

remembering that

$$P b - \left(P + \frac{Q}{2}\right) a \left\{\cos \gamma + \frac{b \sin \gamma}{m}\right\} = 0$$

The only term that does not divide through by $\cos \alpha$ is

$$- \frac{R a}{2}\left(\cos \gamma + \frac{b \sin \gamma}{m}\right) \sin \alpha,$$

and w cannot therefore be constant, because $\cos \gamma + \frac{b \sin \gamma}{m}$

cannot be zero, for if it were zero, then Pb would also be zero, which is impossible; so that w cannot be constant, and in the same way we can show that v cannot be constant; so that this governor is not practically isochronous.

WATT'S GOVERNOR.

Fig. 178 is a diagrammatic representation of this governor, showing the symbols employed for the various forces and dimensions. For the sake of simplicity it is assumed that $AE = DF = m$, and $AC = CD$, that the weights of and the centrifugal force acting on the arms AB and links DC are small enough to be neglected; and that m is to be considered positive when the point of suspension is on the same side of the spindle G G as the balls, and negative when the governor is cross-armed; and lastly, that the suffixes 0, 1, 2 refer to the middle, lowest, and highest points of the sleeve.

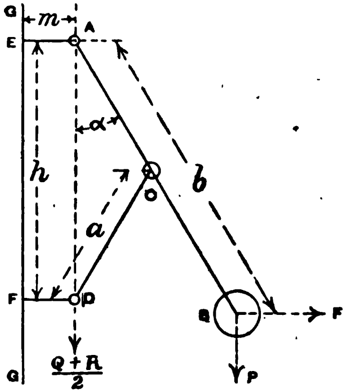


FIG. 178.

Then, from geometrical considerations—

$$h_1 = 2\,a \cos \alpha_1 \quad \ldots \quad (1)$$

$$h = 2\,a \cos \alpha \quad \ldots \quad (2)$$

$$y = h_1 - h = 2a\,(\cos \alpha_1 - \cos \alpha) \quad \ldots \quad (3)$$

whence $\delta y = 2a \sin \alpha\, \delta \alpha \quad \ldots \quad (4)$

and from the principle of virtual velocities

$$F\,b\;\delta\alpha \cos \alpha - P\,b\,\delta\alpha \sin \alpha - \left(\frac{Q}{2} \pm \frac{R}{2}\right)\delta y = 0 \quad . \; . \quad (5)$$

but $F = w^2 \frac{P}{g}(b \sin \alpha + m)$ if R be positive . . . (6)

$F = v^2 \frac{P}{g}(b \sin \alpha + m)$ if R be negative . . . (7)

$F = u^2 \frac{P}{g}(b \sin \alpha + m)$ if R be zero . . . . . . (8)

Hence from (4) and (5)

$$F b \cos \alpha - P b \sin \alpha - (Q + R) a \sin \alpha = 0$$

$$w^2 \frac{P}{g}(b \sin \alpha + m) b \cos \alpha = \sin \alpha [P b + (Q + R) a].$$

Let

$$A = \frac{\tan \alpha}{\sin \alpha + \frac{m}{b}}$$

then

$$\frac{w^2 b}{g} = A\left(1 + \frac{a}{b}\frac{Q}{P} + \frac{a}{b}\frac{R}{P}\right) \quad . \; . \; . \quad (9)$$

$$\frac{v^2 b}{g} = A\left(1 + \frac{a}{b}\frac{Q}{P} - \frac{a}{b}\frac{R}{P}\right) \quad . \; . \; . \; . \quad (10)$$

$$\frac{u^2 b}{g} = A\left(1 + \frac{a}{b}\frac{Q}{P}\right) \quad . \; . \; . \; . \; . \; . \; . \quad (11)$$

are three equations giving the three curves on the governor diagram.

Suppose now it be required to make the governor "quasi-isochronous"—*i.e.*, so that the speeds u_1, u_2, corresponding to the highest and lowest positions of the sleeve, shall be equal; then if A_1, A_0, A_2 be the values of A when α becomes $\alpha_1, \alpha_0, \alpha_2$, the fulfilment of the condition $u_1 = u_2$ requires that

$$A_1\left(1 + \frac{a}{b}\frac{Q}{P}\right) = A_2\left(1 + \frac{a}{b}\frac{Q}{P}\right) \quad . \; . \; . \; . \; . \; . \; . \quad (12)$$

or $A_1 = A_2$; *i.e.*,

$$\frac{\tan \alpha_1}{\sin \alpha_1 + \frac{m}{b}} = \frac{\tan \alpha_2}{\sin \alpha_2 = \frac{m}{b}} \quad . \; . \; . \; . \; . \; . \quad (13)$$

We may choose two of the three quantities α_1, α_2, $\frac{m}{b}$ arbitrarily, and calculate the third; hence there are two problems. First, having given α_1, α_2, to find $\frac{m}{b}$, and, secondly, having chosen α_1 and $\frac{m}{b}$ to find α_2. Taking the former, let us assume that

$$\alpha_1 = 30 \text{ deg.}, \alpha_2 = 45 \text{ deg.}$$

From (13),

$$\frac{\tan 30}{\sin 30 + \frac{m}{b}} = \frac{\tan 45}{\sin 45 + \frac{m}{b}}$$

$$\frac{m}{b}(\tan 30 - \tan 45) = \sin 30 \tan 45 - \sin 45 \tan 30$$

$$\frac{m}{b} = \frac{\frac{1}{2} - \frac{1}{\sqrt{6}}}{\frac{1}{\sqrt{3}} - 1} = - \cdot 22 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

so that m is negative, and the governor will be cross-armed.

Let the values of u be now determined, corresponding to the lowest, highest, and middle positions of the sleeve.

From (11) it appears that u is proportional to $\sqrt{\mathrm{A}}$; and since $u_1 = u_2$,

$$\therefore \sqrt{\mathrm{A}_1} = \sqrt{\mathrm{A}_2} = 1{\cdot}4285.$$

To determine A_0,

$$\frac{c}{2} = 2\,a\,(\cos 30 \text{ deg.} - \cos \alpha_0)$$

where c is the total rise of the sleeve

$$c = 2\,a\,(\cos 30 - \cos 45)$$

$$\cos \alpha_0 = \frac{\cos 30 \text{ deg.} + \cos 45 \text{ deg.}}{2}$$

$$\alpha_0 = 38 \text{ deg. } {\cdot}8 \text{ min.}$$

$$\sqrt{\mathrm{A}_0} = \sqrt{\frac{\tan \alpha_0}{\sin \alpha_0 + \frac{m}{b}}} = 1{\cdot}4$$

and

$$\frac{u_0}{u_1} = {\cdot}98.$$

The diagram of this governor is shown in fig. 179, but it must not be imagined that if the sleeve is at y below y_0, and the speed increases to w, that the sleeve will fall, because Ω_1 Ω_2 is not a curve of descent. The sleeve will rise rapidly, and will pass a point y^1 at which $w^1 = w$. This type of governor is too sensitive, and requires a spring between E and F; in this case R is not constant, but takes the form

$$R = B(y - y_1) \pm C$$

where B and C are constants, and the positive sign must be taken for rising and the negative for falling, because the spring always opposes upward motion. The condition of "astaticity" is $\omega = \Omega_1 = \omega_2$, and according to the definition of the coefficient of sensibility,

$$w - v = \frac{\omega}{n}\ ;\ \Omega_1 - \omega_1 = \frac{\omega}{n_1}\ ;\ \Omega_2 - \omega_2 = \frac{\omega}{n_2}$$

also

$$\Omega_2 - \omega_1 = \frac{\omega}{K}$$

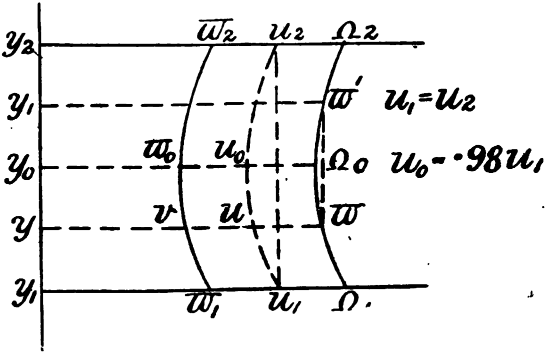


FIG. 179.

where K = 10, so that the extreme variation is one-tenth of the mean speed, whence

$$\frac{1}{K} = \frac{1}{n_1} + \frac{1}{n_2}$$

$$\frac{\omega_1}{\omega} = 1 - \frac{1}{n_1}$$

$$\frac{\Omega_2}{\omega} = 1 + \frac{1}{n_2}$$

$$\therefore \frac{\Omega_2}{\omega_1} = \frac{n_1 - 1}{n_2 + 1} \cdot \frac{n_1}{n_2}$$

But $n_1 = n_2$ very nearly, so that

$$\frac{\Omega_2}{\omega_1} = \frac{2\,K + 1}{2\,K - 1} = \frac{21}{19} \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

and

$$\frac{\Omega_1}{\omega_2} = 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

We must combine these with the equations of equilibrium (9) and (10)

$$\omega_1{}^2 \frac{b}{g} = A_1 \left(1 + \frac{a}{b}\,\frac{Q}{P} - \frac{a}{b}\,\frac{R}{P}\right) \quad . \quad . \quad . \quad (17)$$

$$\Omega_1{}^2 \frac{b}{g} = A_1 \left(1 + \frac{a}{b}\,\frac{Q}{P} + \frac{a}{b}\,\frac{R}{P}\right) \quad . \quad . \quad . \quad (18)$$

$$\omega_2{}^2 \frac{b}{g} = A_2 \left(1 + \frac{a}{b} \cdot \frac{Q}{P} - \frac{a}{b}\,\frac{R}{P}\right) \quad . \quad . \quad . \quad (19)$$

$$\Omega_2{}^2 \frac{b}{g} = A_2 \left(1 + \frac{a}{b}\,\frac{Q}{P} + \frac{a}{b}\,\frac{R}{P}\right) \quad . \quad . \quad . \quad (20)$$

whence

$$\frac{A_2}{A_1} = \frac{\Omega_2{}^2}{\Omega_1{}^2} = \frac{\Omega_2}{\Omega_1}\,\frac{\omega_2}{\omega_1} = \frac{\Omega_2}{\omega_1}$$

since $n_1 = n_2$, and $\Omega_1 = \omega_2$

$$\therefore A_2 = M\,A_1 = \frac{21}{19} = A_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

$$\therefore \frac{m}{b} = -\,{\cdot}1348.$$

If now we take $\alpha_1 = 30$ deg., $\alpha_0 = 38$ deg. 8 min., $\alpha_2 =$ 45 deg., then

$$\sqrt{A_1} = 1{\cdot}2573$$

$$\sqrt{A_0} = 1{\cdot}2748$$

$$\sqrt{A_2} = 1{\cdot}3211$$

and

$$\frac{u_0}{u_1} = 1{\cdot}0143 \quad \frac{u_2}{u_1} = 1{\cdot}0513$$

The curve giving the values of u is shown in fig. 180, and is therefore free from the defects of the "quasi-isochronous" governor.

We may next consider the case in which we assume α_1 and $\frac{m}{b}$ and calculate α_2.

Let us assume $m = 0$, $\alpha_1 = 30$ deg., and calculate α_2, so that the governor may be astatic.

From (21), $\dfrac{\cos \alpha_1}{\cos \alpha_2} = \dfrac{21}{19}$ which gives α_2.

Now, if $K = 10$, and Watt's governor be made astatic with $m = 0$, then the diagram will remain the same whatever be the value given to α_1. For if $\dfrac{c}{q}$ be the height of the sleeve when the arms are expanded to any angle α,

$$\frac{c}{q} = 2a(\cos \alpha_1 - \cos \alpha) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

$$c = 2a(\cos \alpha_1 - \cos \alpha_2) \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

$$\therefore \frac{\cos \alpha}{\cos \alpha_1} = q - 1 + \frac{\cos \alpha_2}{\cos \alpha_1} = q - 1 + \frac{2K - 1}{2K + 1}$$

$$= \frac{q(2K + 1) - 2}{q(2K + 1)}.$$

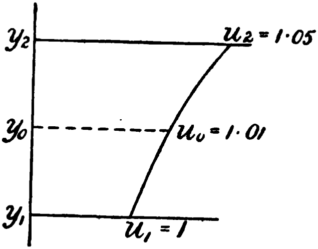


FIG. 180.

Also from (11),

$$u^2 = \frac{1}{\cos \alpha} \frac{g}{b} \left(1 + \frac{a}{b} \frac{Q}{P}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

$$u_1^2 = \frac{1}{\cos \alpha_1} \frac{g}{b} \left(1 + \frac{a}{b} \frac{Q}{P}\right)$$

$$\therefore \frac{u}{u_1} = \sqrt{\frac{q(2K + 1)}{q(2K + 1) - 2}}$$

which is independent of α_1, so that the same diagram applies to every Watt governor having the point of suspension in the axis of revolution, provided $K = 10$, and the choice of $\alpha_1, \dfrac{a}{b}, \dfrac{Q}{P}$ is wholly unrestricted, and may be made to suit the convenience of the engineer. The formula applies equally to the Porter governor, which is only a particular form of Watt's. Watt's governor, with equal arms A C, C D, possesses another useful property, namely, that the total weight reduced to the sleeve is constant in every position of the latter. By the expression, "weight reduced to the sleeve," is meant the weight of a single mass which, if concentrated at the centre of the sleeve, and submitted to the same forces which act upon the real masses, would have the same acceleration as the sleeve has when connected to the actual weights. It may be calculated by equating its virtual velocity to the virtual velocities of the real weights; thus, let Π be the total weight of the governor reduced to the sleeve—*i.e.*, the real weight Q plus a weight 2 X, which would have the same virtual moment as the sum of the virtual moments of the other moving parts; then

$$X \, \delta y = P b \, \delta \alpha \sin \alpha,$$

$$\text{and } \delta y = 2 a \sin \alpha \, \delta \alpha;$$

$$\text{whence } \Pi = Q + 2X = Q + \frac{b}{a} P = \text{constant} \quad . \quad . \quad (25)$$

If H be the height of the cone of revolution of the balls in feet, then

$$H = b \cos \alpha, \text{ since } b \text{ is in feet};$$

$\therefore$ from (24),

$$H = \frac{g}{u^2}\left(1 + \frac{a}{b}\frac{Q}{P}\right)$$

$$= \frac{g}{w^2}\left(1 + \frac{a(Q + R)}{b P}\right)$$

$$= \frac{g}{v^2}\left(1 + \frac{a(Q - R)}{b P}\right)$$

$$\therefore \frac{v - w}{u} = \frac{\sqrt{1 + \frac{a(Q + R)}{b P}} - \sqrt{1 + \frac{a(Q - R)}{b P}}}{\sqrt{1 + \frac{a}{b}\frac{Q}{P}}}$$

$$= \frac{\sqrt{b P + a Q + a R} - \sqrt{b P + a Q - a R}}{\sqrt{b P + a Q.}}$$

$$= \frac{N_1 - N_2}{N}$$

where N_1, N_2 are the numbers of revolutions per minute at which the sleeve will rise and fall, the number of revolutions when friction does not act being N.

COMMON FORM OF GOVERNOR.

The form of governor shown in outline in fig. 181 has its axis horizontal or vertical; G H is this axis, around which D rotates at a radius K D = d; the point F moves a sleeve along the axis towards G when the ball moves outwards. We shall only consider the case in which the arms E D, D F are at right angles. There is a spring which is compressed when F moves towards G. Let 2 S be the force of the spring, and α_1 the greatest value of α; then

$$S = A + B b (\sin \alpha_1 - \sin \alpha)$$

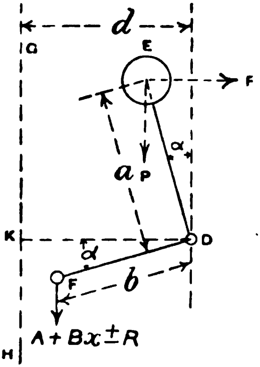


FIG. 181.

where A and B are constants, and the resistance at F parallel to the axis is S + R or S − R, according as motion is towards G or H.

First, let the axis of revolution be horizontal, and let F_1 be the centrifugal force of one ball; then

$$F_1 \, a \cos \alpha = S b \cos \alpha$$

$$\therefore a \frac{P \omega^2}{g} (d - a \sin \alpha) = S b,$$

assuming that the speed is sufficiently great to make the effect of gravity negligable. Firstly, suppose that the governor is to be made isochronous.

$$\frac{P\,\omega^2}{g}(d - a\sin\alpha)\,a = b\,\{A + B\,b\sin\alpha_1 - B\,b\sin\alpha\}$$

and ω will be constant if

$$\frac{P\,\omega^2\,d\,a}{g} = b\,A + B\,b^2\sin\alpha_1$$

and $$\frac{P\,\omega^2\,a^2}{g} = B\,b^2.$$

Supposing all the other quantities are assumed, these are two equations to find A and B—that is to say, the spring is one which exerts a force $\frac{1}{6}$ B for a compression of 1 in., or which would exert a force 2 B for a compression of 1 ft., and which exerts a pressure 2 A when the sleeve is in its lowest position. An isochronous governor is far too sensitive, and would hunt; it is better therefore to make it astatic—*i.e.*,

$$\Omega_1 = \omega_2 = \omega$$

$$\frac{P\,\Omega_1{}^2}{g}(d - a\sin\alpha_1)\,a = b\,\{A + R + B\,b\sin\alpha_1 - B\,b\sin\alpha_1\}$$
$$= b\,\{A + R\}$$

$$\frac{P\,\omega_2{}^2}{g}(d + a\sin\alpha_1)\,a = b\,(A - R + 2\,B\,b\sin\alpha_1).$$

Adding $$\frac{P\,\omega^2\,d\,a}{g} = A\,b + B\,b^2\sin\alpha_1$$

Subtracting $$\frac{P\,\omega^2\,a^2\sin\alpha_1}{g} = b\,R - B\,b^2\sin\alpha_1.$$

From the above equations, if R, P, ω, a, d, and α_1 be assumed, A and B may be calculated.

When the axis of revolution is vertical, the general equation takes the form—

$$\frac{P\,u^2}{g}(d - a\sin\alpha)\,a - P\,a\sin\alpha$$
$$= b\,\{A + B\,b\,(\sin\alpha_1 - \sin\alpha)\}$$

and when u is changed to v or w, A must be altered to A − R or A + R;

$$\therefore\ \frac{P\,\Omega_1{}^2\,(d - a\sin\alpha_1)\,a}{g} - P\,a\sin\alpha_1 = b\,(A + R)$$

whence, if R be assumed, A can be found.

$$\frac{P\,\omega_2{}^2}{g}\,(d + a \sin \alpha_1)\,a + P\,a \sin \alpha_1$$
$$= b\,(A - R) + 2\,B\,b^2 \sin \alpha_1,$$

from which we may calculate B.

Fig. 182 is the outline of Proell's governor, the axis of revolution being G G. The numerical values chosen for the various symbols shown on the figure were measured from a governor actually in operation, and are as follow :—

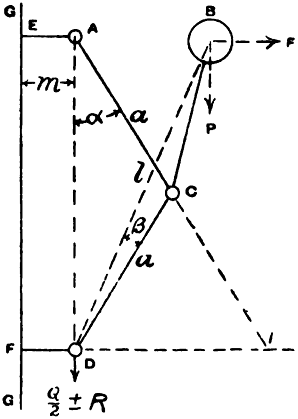


FIG. 182.

d, the diameter of the ball B, $= 1{\cdot}562\,c$, where c is the travel of the sleeve.

$$\frac{a}{l} = {\cdot}693 \quad \frac{m}{l} = \frac{1}{6} \quad \frac{c}{l} = \frac{4}{15}$$

$$= 9^\circ . 43', \ \alpha_1 - \beta = 20^\circ . 21', \ \alpha_0 - \beta = 30^\circ, \ d_2 - \beta = 37^\circ . 59'$$

The instantaneous centre of motion of the arm B C D is I, and the conditions of equilibrium are expressed by equating the sum of the moments of the forces about I to zero ; th[illegible]

$$\frac{u^2\,l}{g} = A + B\,\frac{Q}{P}$$

in which
$$A = \frac{\frac{2a}{l}\sin\alpha - \sin(\alpha - \beta),}{\left[\frac{m}{l} + \sin(\alpha - \beta)\right]\cos(\alpha - \beta).}$$

$$B = \frac{\frac{a}{l}\sin\alpha}{\left[\frac{m}{l} + \sin(\alpha - \beta\right]\cos(\alpha - \beta).}$$

In order to ensure that the sleeve shall move in the proper direction throughout the whole of its travel, we must have $u_2 > u_0 > u_1$; or

$$A_2 + B_2\frac{Q}{P} > A_0 + B_0\frac{Q}{P} > A_1 + B_1\frac{Q}{P}$$

and, therefore, $\frac{Q}{P} > 1{\cdot}08$.

If Q be taken = 2·4 P, as in the case of the governor before mentioned, the values of u will be

$$\frac{u_2}{u_0} = 1{\cdot}03 \quad \frac{u_1}{u_0} = {\cdot}987.$$

The curve at its lower part, therefore, approaches that of isochronism too nearly to be satisfactory.

To make the governor astatic, we must have $\omega_2 = \Omega_1$, so that

$$A_2 + B_2\frac{Q}{P} - B_2\frac{R}{P} = A_1 + B_1\frac{Q}{P} + B_1\frac{R}{P};$$

and if $K = 10$,

$M = \frac{21}{19}$, as in the case of Watt's governor; also astaticity requires

$$A_2 + B_2\frac{Q}{P} + \frac{B_2 R}{P} = M^2\left(A_1 + B_1\frac{Q}{P} - B_1\frac{R}{P}\right)$$

The last two equations give us—

$$\frac{Q}{P} = 3{\cdot}628 \text{ and } \frac{R}{P} = {\cdot}226,$$

and the mean speed ω may be calculated from the equation

$$\frac{\omega^2}{g}l = A + B\frac{Q}{P}$$

for any given value of l.

CHAPTER XVIII.

SPRINGS.

IN fig. 183 is a conical spiral upon which a compressive force W acts along the axis of the cone. This is equivalent to a couple Wr at the point A, r being the radius at A and a force W at A. Now, the line of action of W is not parallel to a section normal to the centre line of the spiral. In fig. 130 the couple Wr is represented by a line perpendicular to its plane, and directed so that the turning appears clockwise when looking along the line in the direction of the arrow. This can be replaced by two components perpendicular to one another, viz., $Wr \sin \psi$, which tends to twist the spiral counter clockwise, and $Wr \cos \psi$ tending to bend the spiral so as to diminish its radius of curvature; also W can be resolved into a shearing force $W \sin \psi$, and a thrust $W \cos \psi$. These two last may be neglected, and in most cases ψ is so nearly a right angle that the bending moment may be neglected, so that the twisting moment may be taken as Wr.

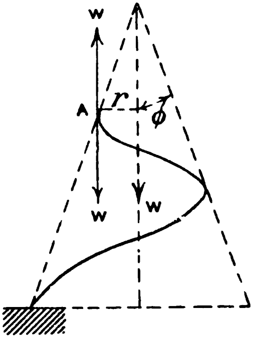


FIG. 183.

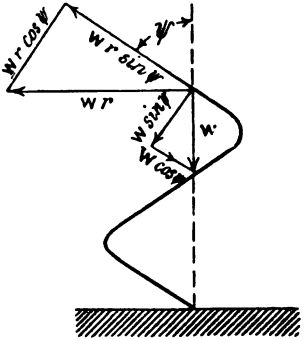


FIG. 184.

Let δ be the deflection of a conical spiral of uniform section under a load W; the greatest and least radii of the spiral are R_n, R_o; I is the polar moment of inertia of the section, C is a constant depending on the form of section, G is the transverse modulus of elasticity, l the increment of radius per coil; then

$$\delta = \frac{\pi C W}{2 l G I}(R_n^4 - R_o^4)$$

Circular Section.—For a circular section $C = 1$, $I = \frac{\pi d^4}{32}$, so that

$$\delta = \frac{16 W}{l G d^4}(R_n^4 - R_o^4)$$

and the stress

$$f = \frac{16 W R_n}{\pi d^3}$$

If the diameter of the wire decreases with the radius, so that

$$d \propto \sqrt[3]{R}$$

the spring has equal strength at every section. If $d^3 = M R$

$$\delta = \frac{24 W}{G l M^{\frac{4}{3}}}\left(R_n^{\frac{8}{3}} - R_o^{\frac{8}{3}}\right)$$

and the stress is

$$f^1 = \frac{16 W^1}{\pi M}$$

so that the greatest safe deflection is

$$\delta^1 = \frac{1{\cdot}5\, \pi f^1}{G l M^{\frac{1}{3}}}\left(R_n^{\frac{8}{3}} - R_o^{\frac{8}{3}}\right)$$

and the greatest safe load is

$$W^1 = \frac{M \pi f^1}{16} = {\cdot}196\, M f^1$$

For a given length of spring and weight of material a circular section gives the least deflection.

Elliptical Section.—This section is used for buffer springs, the larger diameter D being parallel to the axis of the spring, and the smaller d at right angles thereto.

$$C = \frac{(D^2 + d^2)^2}{4 D^2 d^2}$$

$$I = \frac{\pi D d (D^2 + d^2)}{64}$$

so that for a uniform section

$$\delta = \frac{8 W}{l G} \, \frac{(D^2 + d^2)}{D^3 d^3} (R_n^4 - R_o^4).$$

If the coils just fit so that $l = d$, then

$$\delta = \frac{8\,W}{G}\;\frac{D^2 + d^2}{D^3\,d^4}\,(R_n{}^4 - R_o{}^4).$$

The greatest stress in the material is given by

$$f = \frac{16\,W\,R}{\pi\,D^2\,d^2}$$

and is found at the extremities of the shorter diameter. If d^2 varies as R, the stress will be the same at every section. Let $d^2 = M\,R$, then

$$\delta = \frac{64\,W}{G\,D\,l\,\sqrt{K}}\left\{\frac{1}{5\,K}\left(R_n{}^{\frac{5}{2}} - R_o{}^{\frac{5}{2}}\right) + \frac{1}{7\,D^2}\left(R_n{}^{\frac{7}{2}} - R_o{}^{\frac{7}{2}}\right)\right\}$$

Rectangular Section.—This is a very common form of section, especially in buffer springs, &c., where the coils overlap. If the longer side of the rectangle is D, and the shorter is d, then in beams subject to torsion

$$f = a\,.\,G\,.\,d\,.\,\theta\,;\ T = b\,G\,D\,d^3\,\theta = c\,f\,D\,d^2.$$

Where f is the greatest stress produced by a couple T, a, b, c are coefficients given in the table, and θ is the angle of torsion per unit of length reckoned in radians $\left(1 \text{ radian} = \frac{180 \text{ deg.}}{\pi}\right)$

When one side of the rectangle is more than four times the

Ratio of sides $\frac{D}{d}$	a	b	c
1	·675	·141	·208
2	·930	·229	·246
3	·985	·263	·267
4	·997	·280	·282
5	·999	·291	·291
6	1·000	·298	·298
10	1·000	·312	·312
20	1·000	·323	·323
100	1·000	·331	·331
∞	1·000	·333	·333

length of the other, the formula $f = G\,\theta\,d$ may be taken without sensible error, and if D is more than $10\,d$,

$$T = \tfrac{1}{3}\,f\,D\,d^2 \text{ very approximately.}$$

For a spiral spring of rectangular section for all ratios of sides commonly met with (except when $D = d$),

$$\frac{I}{C} = \frac{D\,.\,d\,.\,^{3}}{3},$$

so that for a spring of uniform section

$$\delta = \frac{3\,\pi\,W}{2\,l\,G\,D\,d^3}\,(R_n{}^4 - R_o{}^4);$$

or if the coils just fit one another so that $l = d$,

$$\delta = \frac{3\,\pi\,W}{2\,G\,D\,d^4}\,(R_n{}^4 - R_o{}^4).$$

Also,

$$f = \frac{3\,W\,R_n}{D\,d^2}$$

so that the greatest safe deflection is found to be—

$$\delta^1 = \frac{\pi\,f}{2\,G\,d^2}\,.\,\frac{(R_n{}^4 - R_o{}^4)}{R_n}$$

where f and W are the greatest safe stress and load. In practice D is usually constant, and c diminishes with the radius. When d^2 is proportional to R, the maximum stress is constant. If $d^2 = M\,R$,

$$\delta = \frac{{\cdot}479\,W}{G\,l\,D\,M^{\frac{3}{2}}}\,(R_n{}^{\frac{5}{2}} - R_o{}^{\frac{5}{2}}).$$

It is, however, most convenient and sufficiently accurate to take the mean value of d, and to use the formula for uniform section.

Square Section.—For a conical spiral of uniform section,

$$\delta = \frac{3{\cdot}54\,\pi\,W}{l\,G\,s^4}\;(R_n{}^4 - R_o{}^4).$$

$$f = \frac{4{\cdot}79\,W\,R_n}{s^3}$$

For hardened steel G may be taken as 5,600 tons per square inch.

Cylindrical Spirals: Circular Section.—In this case

$$\delta = \frac{64\,n\,W\,R^3}{G\,d^4}$$

where n is the number of coils.

The greatest stress is

$$f = \frac{16\, \mathrm{W\, R}}{\pi\, d^3}.$$

Elliptical Section.—

$$\delta = \frac{32\, n\, \mathrm{W\, R}^3 . (\mathrm{D}^2 + d^2)}{\mathrm{G\, D}^3\, d^3}$$

$$f = \frac{16\, \mathrm{W\, R}}{\pi\, \mathrm{D}\, d^2}.$$

Rectangular Section $D > 6\, d$.—

$$\delta = \frac{6\, n\, \pi\, \mathrm{W\, R}^3}{\mathrm{G\, D}\, d^3};$$

$$f = \frac{3 . \mathrm{W} . \mathrm{R} .}{\mathrm{D}\, d^2}.$$

Square Section.—

$$\delta = \frac{2 . \pi . n . \mathrm{R}^3 . \mathrm{W}}{\cdot 141 . \mathrm{G} . s^4};$$

$$f = \frac{4\ 79\, \mathrm{W\, R}}{s^3}.$$

Mr. Hartnell has experimented on springs such as are used for governors and safety valves, and gives 60,000 lb. and 70,000 lb. per square inch as the safe stress for ⅜ in. wire, and 50,000 lb. for ½ in. wire. He finds that G varies from 13,000,000 lb. for ¼ in. wire to 11,000,000 for ⅜ in. wire. When the diameter of wire is less than ⅜ in., he gives the maximum load—

$$\mathrm{W} = 12000\, \frac{d^3}{\mathrm{R}}$$

and

$$\delta = \frac{\mathrm{W}\, n\, \mathrm{R}^3}{180000\, d^4}.$$

The Board of Trade rules for the springs of safety valves are—

$$d = \left(\frac{\mathrm{W\, D}}{\mathrm{C}}\right)^{\frac{1}{3}}.$$

$$\delta = \frac{\mathrm{W\, D}^3\, n}{\mathrm{E}\, d^4}$$

where D = the diameter of spiral in inches
n = number of coils
W = the whole pressure on the valve in pounds
C = 8,000 for round wire and 11,000 for square wire
E = 30 for square wire and 22·8 for round

d is the diameter of the wire in inches in the first formula, and sixteenths in the second.

The formulæ used by Messrs. Armstrong, Mitchell, and Co. are—

$$W = \frac{35000\, d^3}{D}$$

$$\delta \text{ for load } W = \frac{\cdot 02333\, n\, D^2}{d}.$$

The first equation may be written in the form—

$$W\,\frac{D}{2} = \frac{\pi}{16}\, d^3 \times 89250$$

i.e., $$f = 89250 \text{ lb.};$$

and the second may be written thus—

$$\delta = \frac{W\, n \left(\frac{D}{2}\right)^3}{188000\, d^4}$$

so that $$\frac{G}{64} = 188000 \text{ lb.}$$

$$G = 12000000 \text{ lb.}$$

$$= 5400 \text{ tons nearly.}$$

For brass springs the above firm give—

$$d = \left(\frac{W D}{10000}\right)^{\frac{1}{3}}$$

and $$\delta = \cdot 0136\, n\, \frac{D^2}{d}$$

for the greatest safe load W.

NUMERICAL EXAMPLES.

Example I.—In a Ramsbottom safety valve there are two valves, each 3 in. in diameter, and the pressure at which the valve blows off is 160 lb. per square inch. The mean diameter of the spiral is 3½ in., and the diameter of the wire is $\frac{13}{16}$ in. There are six turns, and the full lift is $\frac{1}{4}$ in. What is the stress per square inch before lifting and the initial extension of the spring, and what is the greatest extension and the maximum stress.

Let f = stress before lifting

$$= \frac{16\,\mathrm{W\,R}}{\pi\,d^3}$$

$$= \frac{16 \times 2 \times {\cdot}7854 \times 3^2 \times 1\tfrac{3}{4} \times 160}{\pi \times ({\cdot}8125)^3}$$

$$= 37600$$

δ = initial extension

$$= \frac{64\,n\,\mathrm{W\,R}^3}{\mathrm{G}\,d^4}$$

Let $\frac{\mathrm{G}}{64} = 180000$ lb.

$$\delta = \frac{160 \times 2 \times {\cdot}7854 \times 3^2 \times 6\,(1{\cdot}75)^3}{180000 \times ({\cdot}8125)^4}$$

$$= {\cdot}925 \text{ in.}$$

∴ greatest extension = 1·175 in., and greatest stress = $f \times \frac{1{\cdot}175}{{\cdot}925} = 4775$.

Example II.—Each ball of a vertical governor of the type shown in fig. 181 weighs 5·8 lb., and at 180 revolutions per minute revolves in a circle of $4\frac{7}{8}$ in. radius, the arms carrying the balls being then vertical, and those that lift the sleeve horizontal. The former are $4\frac{1}{4}$ in. long and the latter $3\frac{1}{4}$ in. The lift of the governor at 189 revolutions is $1\frac{1}{2}$ in. To design a suitable spring.

If ω is the angular velocity, r the radius of revolution in feet, and W the weight of the two balls, the force exerted on the sleeve by their centrifugal force, at 189 revolutions per minute, is—

$$\mathrm{F}_1 = \frac{\mathrm{W}}{g} r\,\omega^2 \times \frac{4\frac{1}{4}}{3\frac{1}{4}} = \frac{11{\cdot}6}{32} \times \left(\frac{2\,\pi \times 189}{60}\right)^2 \times \frac{6{\cdot}835}{12} \times \frac{4\frac{1}{4}}{3\frac{1}{4}}$$

the radius of revolution being readily shown to be 6·835 in.

∴ $\mathrm{F}_1 = 106$ lb.

The additional force exerted by the weight of the balls is—

$$\mathrm{F}_2 = 11{\cdot}6 \times \frac{1{\cdot}96}{2{\cdot}88} = 7{\cdot}9 \text{ lb.}$$

since 1·96 in. is the overhang of the balls, and 2·88 in. is the horizontal projection of the inner arm.

The total force F on the spring is therefore 113·9 lb.

Let R be the radius of the spring, and d the diameter of the wire; then

$$d^3 = \frac{\text{W} . \text{R}}{\cdot 196\, f}$$

$$= \frac{113{\cdot}9\ \text{R}}{11760} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

When the number of revolutions is 180, and the arms are vertical, the force on the spring is

$$\text{F}_3 = \frac{11{\cdot}6}{32} \times \left(\frac{180 \times 2\pi}{60}\right)^2 \times \frac{4\frac{7}{8}}{12} \times \frac{4\frac{1}{4}}{3\frac{1}{4}}$$

$$= 68{\cdot}6 \text{ lb.}$$

Hence $\text{F} - \text{F}_3$ can compress the spring 1½ in., using the formula—

$$\delta = \frac{\text{W}\, n\, \text{R}^3}{180000\, d^4}$$

$$1\tfrac{1}{2} = \frac{45{\cdot}3\, n\, \text{R}^3}{180000\, d^4} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

From equations (1) (2) we can find two of the three quantities n, R, d, if we assume one of them. Suppose R = 1⅜ in., then

$$d^3 = \frac{113{\cdot}9 \times 1{\cdot}375}{11760} = {\cdot}0133$$

$$d = {\cdot}237 \text{ in.} = \tfrac{1}{4} \text{ in., say;}$$

and from (2)

$$n = \frac{180000 \times ({\cdot}237)^4 \times 1\frac{1}{2}}{45{\cdot}3 \times (1{\cdot}375)^3}$$

$$= 7{\cdot}2 \text{ turns.}$$

Example III.—A safety valve has a diameter of 3 in., and the pressure is 160 lb.; there are 11 turns in the spiral spring, whose mean diameter is 2¾ in., and it is of square section side d; to find d and the necessary compression from the rules of the Board of Trade given above.

$$d = \left(\frac{\text{W D}}{\text{C}}\right)^{\frac{1}{3}}$$

$$= \left(\frac{160 \times 7{\cdot}0686 \times 2\frac{3}{4}}{11000}\right)^{\frac{1}{3}}$$

$$= {\cdot}656 \text{ in.} = \frac{21}{32} \text{ in.}$$

and $$\delta = \frac{\text{W D}^3 n}{d^4 \text{E}}$$

d being in sixteenths,

$$= \frac{160 \times 7{\cdot}0686 \times (2{\cdot}75)^3 \times 11}{(10{\cdot}5)^4 \times 30}$$

$$= {\cdot}7 \text{ in.}$$

CHAPTER XIX.

AIR VESSELS FOR PUMPS.

WHEN the piston or plunger of a pump is connected to a crank its velocity varies, and so also does that of the discharge. Now, if the pump discharges into a main of any length, and this change of velocity is communicated to the whole mass of water therein, the pressure in the pump and pipes would be enormously increased, and all parts would require to be much stronger, or there would be a breakdown. An air vessel is therefore fitted as close as possible to a pump, so that when the velocity of discharge is above the mean the water may enter the air vessel and compress the air therein, and when the velocity falls below the mean the air vessel may supply the deficiency, so that the velocity in the mains may be unaltered. When the head of water is great, compressed air is supplied to the air vessel to take the place of that absorbed by the water, but when the head is small it is usually sufficient to draw in a small quantity of air at each suction stroke of the pump. The quantity of water that flows into the air vessel may be obtained by calculation, which involves the integral calculus, or graphically in the following manner: It will be clear that the velocity of discharge at any instant is proportional to the piston velocity. Let E F, fig. 186, represent the stroke, and Q P, P C the connecting rod and crank; then the ratio of piston velocity to crank-pin velocity is C H : C P. Next, in fig. 185, take A C, C B each to represent the half revolution, and let

$$\frac{AH}{AC} = \frac{\text{arc } EP}{\text{arc } EPF},$$

and make D H, fig. 185, equal to C H, fig. 186. By making

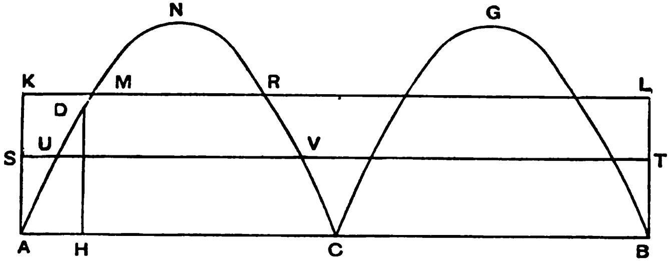


FIG. 185.

this construction for several points, we can set out the curves A D C and C G B. Again the mean velocity of the piston $= \frac{2}{\pi} \times$ the velocity of the crank pin. Let A K represent this velocity, C P representing that of the crank pin, and draw K L parallel to A B. Then the area M N R bears to the area K L B A the same ratio that the quantity

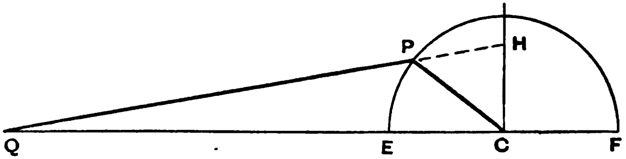


FIG. 186.

absorbed by the air vessel bears to the total discharge per revolution in the case of a double-acting pump. If the pump is single-acting, take A S equal to half A K. Then the area U N V is to the area S T B A as the surplus discharge is to the whole. Similar diagrams may be drawn for pumps having cranks at right angles, or at 120 deg. Fig. 187 is a diagram for two double-acting pumps having cranks at right angles. The curves A D C, C E B are drawn for one pump in the same manner as the curves in fig. 185, and through G and K points bisecting A C, C B. Similar curves are drawn for the other. If the ordinates are then added, the curves F P D, D Q H, &c., are obtained. Then, since there are two pistons, M N is drawn parallel to A B, so that

$$M A = \frac{4}{\pi} \times \text{velocity of crank pin,}$$

and each of the areas cut off above it are proportional to the water discharged into the air vessel, the rectangle M N B A representing the discharge per revolution. The following table gives the percentage of the whole discharge per revolution that enters the air vessel, the obliquity of the connecting rod being neglected.

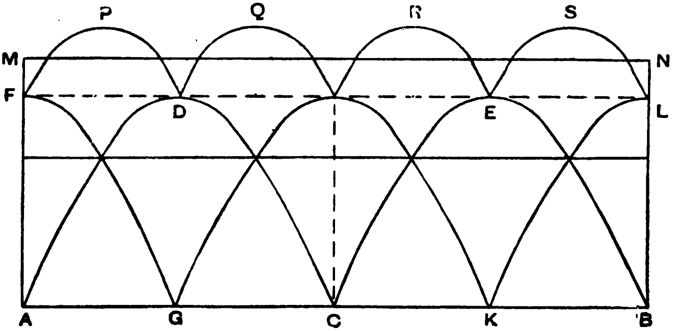


Fig. 187.

Type of Pump.	Excess per cent.
Single-acting pump, one barrel	55
Double-acting pump, one barrel	10·5
Two double-acting pumps, with cranks at right angles	1 05
Two single-acting pumps, with cranks at right angles	35
Three-throw single or double-acting pumps, with cranks at 120 deg.	·58
Five single-acting pumps	·13

Let p be the mean pressure due to the head h against which the pump works, and P the greatest pressure allowed. Let V be the volume of air enclosed in the air vessel, and v

the volume to which it is compressed when the pressure rises from that of the atmosphere to p. To be accurate, V must be measured from the top of the discharge pipe D (see fig. 188). Then—

$$15\,V = p\,.\,v.$$

Let Q^1 be the quantity entering and flowing out of the air vessel in cubic feet.

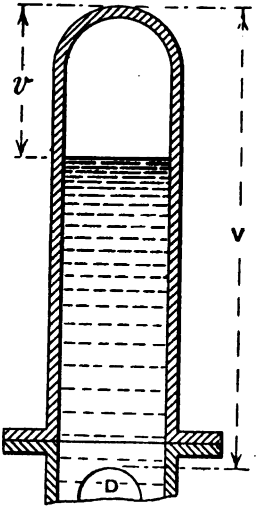


FIG. 188.

$$(v - Q^1)\,P = v\,.\,p$$

$$v = Q^1 + \frac{v\,p}{P}$$

$$v\left(1 - \frac{p}{P}\right) = Q^1$$

$$v = \frac{Q^1\,P}{P - p}$$

$$V = \frac{p\,.\,Q^1\,.\,P}{15\,(P - p)}.$$

If H, h are the heads of water corresponding to pressures P, p,

$$V = \frac{h\,Q^1\,H}{34\,(H - h)}.$$

The greater the variation of pressure allowed in the air vessel—*i.e.*, the greater the fraction $\frac{P - p}{P}$, the smaller will be the volume of the air vessel. Mr. J. Brackenbury, in a paper on "Pumps and Air Vessels," read before the Hull and District Institution of Engineers, states that for a waterworks engine discharging into a long main, where any alteration in the velocity of discharge is attended with great increase of pressure,

$$P - p = \frac{3}{100}\,p,$$

so that $$\frac{P}{P - p} = \frac{103}{3} = 34\tfrac{1}{3}.$$

On the other hand, for a circulating pump delivering against a small head, we may take

$$\frac{P}{P - p} = 2.$$

In the Minutes of the Proceedings of the Institution of Civil Engineers, vol. lxxviii., in a paper on "The Comparative Merits of Vertical and Horizontal Engines, and on Rotative Beam Engines for Pumping," Mr. Rich states that, if provision is not made for breakdown, the volume of the air vessel should be forty times the amount entering and leaving it; but this does not take the head into account, and it is obvious that as this increases the air vessel volume must also increase. If provision is made for a breakdown, he considers that twenty times the amount that enters and leaves the air vessel is sufficient, assuming that one barrel of the pump fails. Under such circumstances, he states that for every 100 gallons discharged per revolution the capacity of the air should be—

For two single-acting pumps, with cranks opposite, 550 gallons; two-throw pumps, 360 gallons; two double-acting pumps, with cranks at right angles, 250 gallons. These

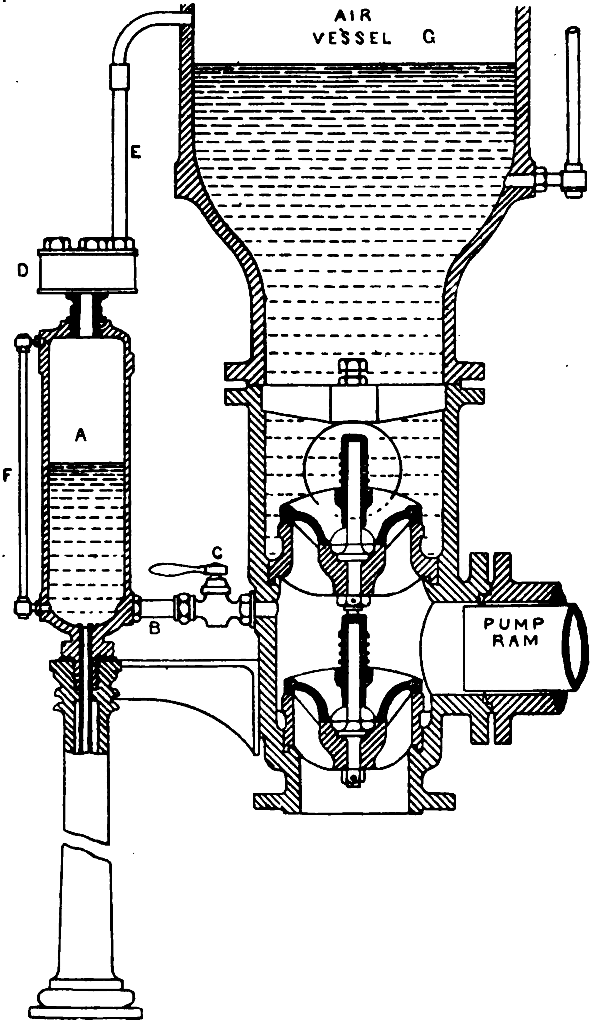


Fig. 189.

volumes should refer to v, the volume of the air in the air vessel, and since

$$v = 20\,Q^1, \text{ and } v = \frac{Q^1\,p}{P - p}$$

$$\therefore \quad \frac{P}{P - p} = 20.$$

$$\text{and} \quad V = \frac{4}{3}\,p\,Q^1.$$

When an air injector is used to supply an air vessel, its volume may be smaller than that obtained by the above calculations, because nearly the whole of the air vessel may be full of air. In this case, if V is the volume of air in the air vessel when the pressure therein is equal to that of the head,

$$(V - Q^1)\,P = V \,.\, p$$

$$V = Q^1 + V\,\frac{p}{P}$$

$$V = \frac{Q^1 \,.\, P.}{P - p}$$

which is independent of the head, but varies with the type of pump and the length of the delivery main. The air-vessel volume must be made larger than V, so that the joint at the bottom may be always covered with water.

Fig. 189 shows Wippermann and Lewis's air injector as applied to a plunger pump. It consists of a cylindrical vessel A, which has no working parts, the water itself forming the piston; at the bottom of the chamber is a small pipe B fitted with a regulating cock C, which is attached to the pump valve box, immediately below the delivery valve. At the top of the vessel is fixed a small gun-metal valve box D, fitted with inlet and outlet air valves, and from this a delivery pipe E communicates directly to the air vessel G.

The action of the apparatus is as follows, viz.: When the main pump draws its water it will partly empty the vessel A, the amount being indicated by the gauge F and regulated by the cock C; on the return of the plunger, the whole of the air drawn into the chamber A will be delivered into the air vessel G, because the pressure in the main pump, when delivering, is in all cases greater than on the suction side.

Another apparatus is Appold's air regulator, fig. 190. The water enters at the bottom on the delivery stroke of the pump, and passes not only through the cock at the centre of the bottom of the air vessel, but also through the valve at its left, so that the cock may be arranged to prevent too rapid an outflow of water, while the entry may be unchecked, it having been found that the same amount of throttling would not serve both for inflow and outflow. It had been found in the first design that if the cock were opened sufficiently, the water would leave the vessel with a rush, and there was then nothing to prevent air entering the pump. Mr. Appold designed an internal balance float which rendered the apparatus automatic. This float was a flat, circular copper tray, lined with gutta-percha, fastened to a tripod or

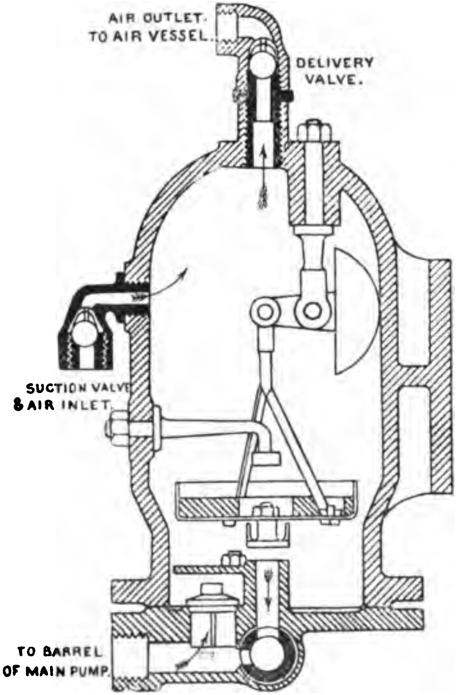


FIG. 190.

yoke, by which it was suspended to the end of a lever of the first order. The other end of the lever carried a counterpoise, which balanced the float. The lever fulcrum was hung from a gun-metal suspension bolt, screwed home through a a boss in the dome of the water vessel. The bottom of the float carried a little flat valve in brass, so that when the water level fell, instead of going away at a gulp into the pump, the outlet was closed, and a certain level was always maintained in the water vessel. The suction and delivery valves are ball valves at the side and top. This description, with fig. 190, is taken from the remarks made by Mr. Amos in the discussion on this paper on air vessels, already referred to.

CHAPTER XX.

ON FLYWHEELS.

SINCE the effort exerted by an engine is variable, and the resistance is, for a time at least, constant, the velocity of rotation would vary considerably at different parts of the revolution, if a flywheel were not used to store and restore the surplus energy.

The energy of the flywheel is $\frac{W V^2}{2 g}$ where W is its weight, and V the mean velocity of its rim in feet per second. Let V_1, V_2 be its greatest and least speeds; then V, the mean velocity, $= \frac{1}{2} (V_1 + V_2)$ very nearly.

$$\text{Let } n = \frac{V_1 - V_2}{V} = \frac{2(V_1 - V_2)}{(V_1 + V_2)}.$$

According to Professor Unwin, the following are suitable values of n :—

	$n =$
Engines doing pumping	1/20
Engines driving machine tools	1/35
Engines driving textile machines	1/40
Engines driving spinning machinery	1/50 to 1/100
Engines driving electric machinery	1/150

Let E be the energy stored and restored by the flywheel; then

$$E = \frac{W(V_1^2 - V_2^2)}{2g}$$

$$= \frac{W(V_1 - V_2)(V_1 + V_2)}{2g}$$

$$= \frac{W^2 n V_2}{g}.$$

$$\frac{W n v^2}{g}$$

The quantity E may be found in the following manner: In fig. 191 is shown the diagram of twisting moment of a single-cylinder double-acting engine AB, BC, each representing half a revolution. The line EF is drawn at a height above AC equal to the mean twisting moment; the area above CD represents energy stored, and that below DQ the

G

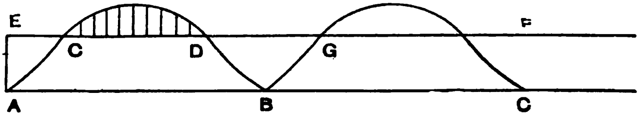


Fig. 191.

energy given out again, and the two are equal to one another. Let T be the mean twisting moment, and let CD be divided into a number—say ten—of equal parts, and the mean ordinate be t; then

$$E = \frac{t.\,CD}{T.\,AB} \times \text{work done per stroke.}$$

In engines having several cranks, or having cylinders whose lines of stroke are not parallel, and which act on one crank, the twisting moment diagram will have several parts per revolution, such as that above CD; only one of these, however, must be taken, as its area represents the energy stored by the flywheel.

The following table gives the weights of flywheels for certain types of engines:—

TABLE OF FLYWHEELS.

Simple Engines.

Type of engine	H S	H S	H S	H S	H S	H S	H S	B P
I.H.P.	..	52½	85½	42	60	500	1000	202
Diameter of cylinder	9	10*	10*	11·8	14½	28*	40	48
Stroke in inches	18	20	20	23·6	30	42	120	108
Revolutions per minute	100	100	100	110	70	90	45	11
Diameter of flywheel in inches	84	84	84	98	132	180	360	324
Boiler pressure	60	50	50	80	80	60	55	35
Normal cut-off in inches	..	5	10	6·6	7½	..	30	20
Weight of flywheel in cwts.	20	34	34	40	56½	130	1090	1000

Compound Engines.

Type of engine	H S A	H S T	H S T	H S T	H S T	H S A	H S T	H S T	B P A	BP A	H P A
I.H.P.	150	90	25	..	1120	1300	900	1200	..	119	224
Diameter of H.P. cylinder	13	13	6	12½	24*	32	27	30	22½	21	30⅜
Diameter of L.P. cylinder	22½	24	10	22	46	60	46	51	45	36	54
Stroke of H.P. piston	24	24	12	30	72	84	66	66	66	66	48
Stroke of L.P. piston	24	24	12	30	72	84	66	66	66	66	48
Revolutions per minute	90	75	150	86	50	50	75	70	22	22	25
Diameter of flywheel in inches	120	108	78	120	360	408	204	240	180	171	188
Boiler pressure	120	80	70	65	80	100	60	90	60	60	80
Normal cut-off in inches	..	12	6	15	18	21	33	33	..	..	16
Weight of flywheel in cwts.	63½	80	10	75	1200	1560	600	900	155	87	125

* Two engines.

In the above table H S stands for horizontal stationary engine, H S A for the same with cranks at right angles, H S T for horizontal stationary tandem, B P for beam pumping engine, B P A for the same with cranks at right angles, and H P A for a horizontal pumping engine having cranks at right angles.

THE COMPENSATING CYLINDERS OF THE WORTHINGTON PUMPING ENGINE.

These may be described in connection with the subject of flywheels, as they act in the same manner, storing up energy when the steam pressure is greater than the resistance, and returning it when through expansion the pressure falls. The Worthington pumping engine has no crank nor flywheel; the piston rod has the piston at one end and the plunger at the other, and there is nothing except the steam pressure to determine the length of stroke. There are always two tandem engines placed side by side. Each engine works its own expansion valves, but the

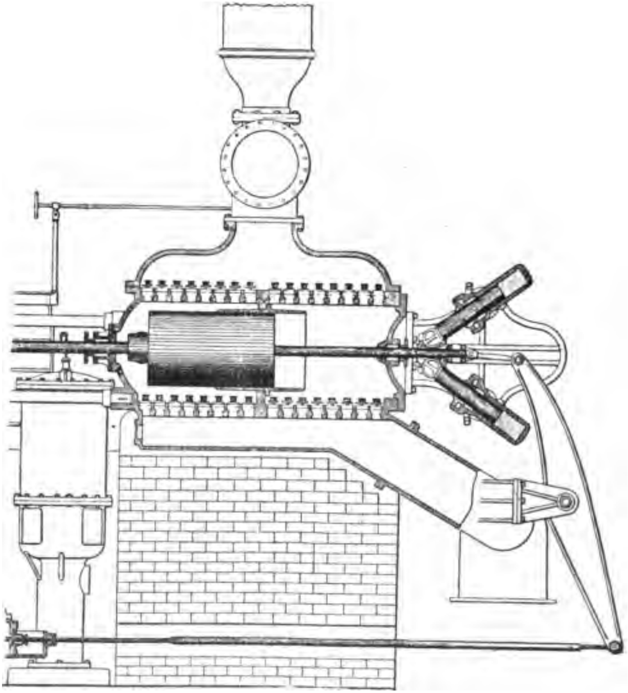


FIG. 192.

distribution valves are actuated by the other. At first these engines were made with very little expansion, so that

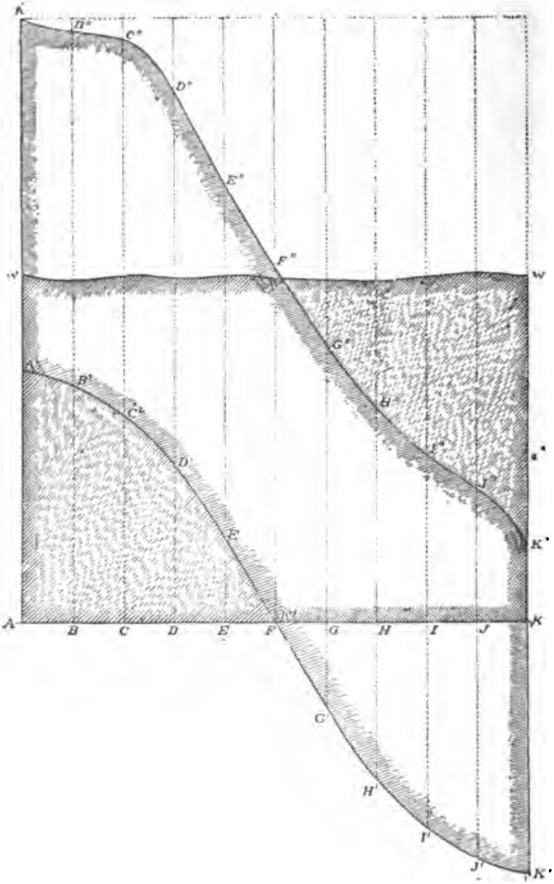


Fig. 193.

the pressure at any point differed very little from the resistance, but under these circumstances they were not economical, and in order to increase the expansion of the steam the compensating cylinders were devised. These consist of plungers working in oscillating cylinders, fig. 192, these being connected to air-compressing pumps or to the delivery main. The plungers are connected to the end of the piston rod, or in some cases they are placed between the steam cylinders and the pump. They exert a force $2\,P \cos \theta$ where P is the pressure on each plunger, and θ the inclination of its axis to the horizontal. This force resists the motion during the first half of the stroke, and assists it during the second.

Fig. 193 shows the diagram of resultant pressure on the pistons A″ B″ K″. W W is the almost constant resistance of the pump, and A′ B′ K′ is the diagram showing the action of the compressors. It will be at once seen how very nearly constant is the combined effort of pistons and compensators. Without the latter the engine cannot finish its stroke, and this is an advantage, especially when pumping oil to a distance, for if a break occurred in the pipes, much oil would be wasted if the engines were not immediately stopped. The sudden fall of pressure prevents the compensators doing their work, and the motion of the pistons ceases.

CHAPTER XXI.

THE DIAGRAM FACTORS OF COMPOUND ENGINES.

IF the cut-off be at $\frac{1}{r}$th of the stroke of an engine, the steam will be expanded r times. Neglecting clearance and the mean absolute pressure, p may be calculated from the formula—

$$p = p_1 \frac{1 + \text{hyp. log. } r}{r},$$

where p_1 is the initial pressure absolute. So that if p_2 is the mean back pressure absolute, the effective pressure p_3 is—

$$p_3 = p - p_2 = p_1 \frac{1 + \text{hyp. log. } r}{r} - p_2.$$

When the compound engine was first introduced, one of the objections to its use was that for a given initial absolute pressure, and a given total number of expansions, the mean pressure referred to the area of the low-pressure piston was generally less than what it would have been had the same number of expansions taken place in the low-pressure cylinder with the same initial absolute pressure; or, to put it in another way, if the combined diagram of the compound engine was made, its area was smaller than the area of an ideal single-cylinder engine having the same expansion and initial pressure. We do not intend to deal with any of the reasons for this apparent loss of power, but shall show to what extent it occurs in different types of engines. By the term diagram factor we mean the ratio of the mean pressure of a compound engine, referred to the area of the low-pressure cylinder to the mean pressure that would have been obtained had the steam expanded hyperbolically in the low-pressure cylinder, with the same number of expansions and the same boiler pressure, taking the back pressure as 3 lb. in a condensing engine, and as 18 lb. in a non-condensing engine.

If we put this mathematically, the diagram factor

$$k = \frac{\text{H.P.} \times 33000}{p_3 \, \text{L A N}};$$

where H.P. = horse power,
L = stroke in feet,
A = area of piston of low-pressure cylinder,
N = number of strokes,
and p_3 is calculated as explained above.

The following numerical examples will make the methods of calculation evident:—

TRIPLE-EXPANSION ENGINES OF THE HISPANIA.

Diameters of cylinders, 20 in., 31 in., and 52 in., with a stroke of 3 ft. Revolutions, 68·93. Horse power, 613. Cut-off at ·47 in high-pressure cylinder. Boiler pressure above atmosphere, 140. The total number of expansions, neglecting clearance, is

$$\left(\frac{52}{20}\right)^2 \times \frac{1}{\cdot 47} = 14\cdot 4 \text{ nearly};$$

$$p = 155 \, \frac{1 + \log. 14\cdot 4}{14\cdot 4} = 155 \times \cdot 254 = 39\cdot 3;$$

$$p_3 = p - p_2 = 39\cdot 3 - 3 = 36\cdot 3$$

$$k = \frac{\text{H.P.} \times 33000}{p_3 \, \text{L A N}}$$

$$= \frac{613 \times 33000}{36\cdot3 \times 3 \times 2123\cdot7 \times 137\cdot86} = \cdot63.$$

WILLANS COMPOUND HIGH-SPEED SINGLE-ACTING NON-CONDENSING ENGINE.

Boiler pressure 135, areas of pistons 71·47, 141·34, stroke 6 in., cut-off in high-pressure cylinder ·336, mean pressure referred to the area of the low-pressure piston 38.

The number of expansions

$$= \frac{141\cdot34}{71\cdot47} \times \frac{1}{\cdot336} = 5\cdot9 \text{ nearly};$$

$$p = 150 \, \frac{1 + \text{hyp. log. } 5\cdot9}{5\cdot9} = 150 \times \cdot47 = 70\cdot5\,;$$

$$p_3 = p - p_2 = 70\cdot5 - 18 = 52\cdot5,$$

p_2 being 18, because the engine is non-condensing.

$$\therefore k = \frac{38}{52\cdot5} = 72.$$

It will be readily seen that, given the horse power, revolutions, boiler pressure, and number of expansions, we can find the volume of the low-pressure cylinder if we know the diagram factor ; after which it will not be difficult to find a suitable diameter and stroke. Then, from other reasons which we cannot go into at present, the cut-off in the high-pressure cylinder may be fixed, from which its volume may also be found. To give an example, suppose we require the diameter of the cylinders of a compound horizontal Corliss mill engine, with cranks at right angles, and are given boiler pressure 90, with 15 expansions, and piston speed 700 ft. per minute, to develop 1,100 horse power, the cylinders being well clothed, but not jacketed. For this we may take $k = \cdot78$.

$$k = \frac{\text{H.P.} \times 33000}{p_3 \, \text{L A N}}\,;$$

$$\therefore \text{A} = \frac{\text{H.P.} \times 33000}{k p_3 \, \text{L N}}\,;$$

$$\therefore \text{A} = \frac{1100 \times 33000}{\cdot 78 \times p_3 \times 700};$$

also

$$p_3 = p_1 \frac{1 + \text{hyp. log. } 15}{15} - 3$$

$$= 105 \times \cdot 247 - 3 = 26 - 3 = 23;$$

$$\therefore \text{A} = \frac{1100 \times 33000}{\cdot 78 \times 23 \times 700} = 2890, \text{ nearly.}$$

so that a diameter of 60$\frac{3}{4}$ in. will be very near the right size.

Let a = area of the high-pressure cylinder, and suppose the cut-off is chosen at 23 per cent of the stroke, then

$$\text{A} = 15 \times \cdot 23\, a;$$

$$\therefore a = \frac{2890}{15 \times \cdot 23} = 840,$$

corresponding to a diameter of 32$\frac{3}{4}$ in., nearly.

In the table below, k is the diagram factor, r is the number of expansions, the pressure p_1 is the absolute pressure in the boiler, or 15 lb. above the working pressure of the boiler, and P.S. means piston speed. J means that jackets are used, and N that they are not used. Where there is any doubt as to whether the engine is condensing or non-condensing, the letters C and N C are used.

TYPE OF ENGINE.	k	r	p_1	P.S.
Pumping Engines—				
Worthington, high duty, J	·86	9·2	74·3	97·5
	·9	13·2	95·4	85
	1	14·1	101·9	86·9
Compound differential, J C	·81	10	68	—
Compound differential, J N C	1	5·45	60	—
	1	4·27	60	—
Moreland's compound rotative vertical, J	1·15	18·2	84	—
Woolf beam, J	·82	9·33	58	195
	1·19	17·4	62	215
Tandem beam, with receiver, J	·93	14	76	263
Leavitt's compound rotative beam, J	·92	14·2	105	260

It will be noticed in the above that k is very high, and lies between ·81 and 1, with moderate expansions, and this is probably due to slow speed and jacketing; with such high values of r as 17·4 and 18·2, k rises to 1·10 and 1·15, which is probably owing to the clearance, which reduces the real number of expansions.

TYPE OF ENGINE.	k	r	p_1	P.S.
Compound Mill Engines, C—				
Corliss horizontal, two cylinders, with cranks at right angles, N	·86	10·3	92	420
Corliss tandem horizontal, N	·76	11·36	95	648
Corliss horizontal, two cylinders, with cranks at right angles, N	·81	15·05	107	703
Compound marine engines, screw steamship Rush, two cylinders, cranks at right angles, J	·75	6·95	87	319
Screw steamship Koning der Neiderland, J, same as above	·7	5·9	74	412
Steamship Lusitania same as above	·67	5·64	70	347
Compound locomotive, Worsdell's Express, N	·64	4·18	175	840
	·85	2·99	185	168
	1·1	4·18	190	126
Russian passenger: Grazi and Tsaritsin Railway, N	·94	7·15	150	323
	·95	4·28	150	258
	·96	2·46	150	97
Russian goods: N	·5	4·07	150	262
	·67	3·19	150	234
	·74	2·66	150	200
	·82	2·33	150	174
Passenger locomotive, Bayonne and Biarritz Railway, N	·716	4·62	165	319
Passenger locomotive, Paris and Orleans Railway, N	·63	2·45	135	745
	·7	3·8	135	686
Willans' vertical tandem, high-speed, single-acting, non-condensing, N	·7	8·47	150	404
	·72	5·9	150	400
	·676	4·2	129	400
	·78	4·2	120	212
	·835	4·2	117	100

In the following table for triple-expansion marine engines, the mean value of the factor from ten examples is ·635.

TYPE OF ENGINE.	k	r	p_1	P.S.
Triple-expansion Marine—				
Steamship Westmoreland, three-throw crank, J	·71	13·1	160	408
Steamship Hispania, same as above	·63	14·4	155	414
Steamship African, same as above	·67	12·6	165	513
Steamship Para, same as above	·74	15·6	165	340
Steamship Shakespeare, same as above	·697	11·6	155	394
Steamship Kaiser Wilhelm, same as above	·53	8·45	172	800
Steamship Arabian, two cranks, tandem, with two H.P. cylinders, N	·60	8·4	165	388
Steamship Claremont, two cranks, semi-tandem, converted	·54	10·5	165	435
Steamship Mariposa, three-throw crank	·615	11·06	175	520
Name unknown, diagrams taken from *The Practical Engineer*, March 18.	·62	18·6	173	486
(In the last three examples no information could be obtained as to jackets.)				
Willans' triple-expansion high-speed non-condensing tandem, N	·69	6·33	167	409
	·71	6·75	187·5	400

The probable value of *k* for a compound marine engine is about ·7; for a Corliss horizontal engine, without jackets, from ·76 to ·86. For the compound locomotive it is difficult to fix any value; nevertheless at a high piston speed of from 700 ft. to 800 ft. per minute, $k = \cdot 63$ or $\cdot 64$; and at a slow speed, say 170 ft. per minute, k increases to about $\cdot 8$. For a given engine, the slower the speed the greater is k, as will be seen in the last three examples of the Willans engine. We believe that this is due to there being less wire-drawing and more time for initial condensation to take place, the subsequent re-evaporation during expansion increasing the mean pressure.

CHAPTER XXII.

METHOD OF DRAWING THEORETICAL DIAGRAMS OF COMPOUND ENGINES.

IN every case which we have seen in which this subject has been treated, theoretical diagrams are drawn in which wire-drawing at cut-off and drop in pressure caused by the resistance of the passages between the cylinders during admission have been neglected, nor is anything said about the manner in which these theoretical diagrams must be modified to obtain from them the probable diagrams. After considerable study of the subject, we have come to the conclusion that it is best to make certain allowances for drop in the receiver and wire-drawing at cut-off during the first calculations, and thus make the theoretical diagrams resemble at once as nearly as possible the probable diagrams. The object of setting out these is to obtain the best proportions of cylinders. For example, in a compound engine with cranks at right angles, it will generally be desirable to obtain equal power in the two cylinders, equal initial stresses, and only the most economical amount of drop in pressure when the steam is exhausted. In our opinion it is the best to first calculate the size of the low-pressure cylinder, assume some point of cut-off in the high-pressure cylinder which, with the assumed number of expansions, will give its magnitude; to then set out the probable diagrams, and if any faults appear in them to correct them by altering the size of the high-pressure cylinder. In designing triple and quadruple expansion engines, the magnitudes of the intermediate-pressure cylinder or cylinders will also have to be calculated, and if necessary altered subsequently. These can readily be calculated from the high-pressure and low-pressure cylinders, in a manner to be explained further on. This takes some little time, but is preferable to the usual theoretical method, because correct instead of incorrect results will be obtained. For example, in drawing the theoretical diagrams of a compound engine, with cranks at right angles, neglecting wire-drawing and drop of pressure in the receiver, we found that for equality of horse power in the two cylinders cut-off in the low-pressure cylinder had to take place after half-stroke; whereas in actual practice it was at about one-third of the stroke, owing to the faults above mentioned. We may here mention that the earlier the cut-off in the low-pressure cylinder, the greater is the power developed therein.

To explain what we believe to be the best method of constructing these diagrams, we shall first take the case of a compound engine with cranks at right angles; the diagrams are shown in fig. 194. The volumes of cylinder and clearance of receiver, and of low-pressure cylinder and clearance, are H c_h, R, L, c_l. The steam is admitted at pressure p_0, which is below the boiler pressure, and is cut off at pressure p_1, which is always less than p_0. because of the wire-drawing that takes place at cut-off. This is least with Corliss valves, because they close rapidly. The expansion curve 12 is a hyperbola; the exhaust in reality begins before the end of the stroke, but for simplicity it may be treated as if it takes place instantaneously at the end, an inclined line, such as 2′3, being afterwards drawn before the mean pressure is calculated. Then

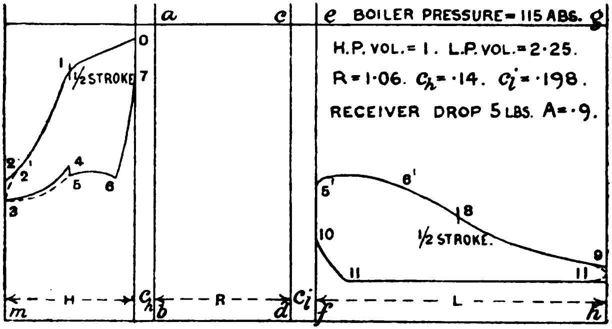

a
c
e
BOILER PRESSURE = 115 ABS.
g
0
1
½ STROKE
7
H.P. VOL. = 1. L.P. VOL. = 2·25.
R = 1·06. c_h = ·14. c_l' = ·198.
RECEIVER DROP 5 LBS. A = ·9.
2′
2
4
5
6
3
5′
6′
8
10
½ STROKE
9
11
11
H
c_h
R
c_l
L
m
b
d
f
h

FIG. 194.

$$p_2 (H + c_h) = p_1 (k H + c_h),$$

where k is the fraction of the stroke at which cut-off takes place. Next assume p_7 and p_{10}, the pressures of compression in the two cylinders. The point of cut-off in the low-pressure cylinder is 8, the fraction of the stroke being k_1, here less than ½, and, if the steam expanded as a perfect gas at constant temperature, we should have

$$p_8 (k_1 \mathrm{L} + c_l) - p_{10} c_l = p_1 (k \mathrm{H} + c_h) - p_7 c_h.$$

Actual diagrams generally show that the quantity on the left-hand side is less than that on the right; this is principally caused by wire-drawing, and the pressure in the cylinder is always less than that in the receiver. For purposes of calculation, we may write

$$p_8 (k_1 \mathrm{L} + c_l) - p_{10} c_l = \mathrm{A} [(p_1 (k \mathrm{H} + c_h) - p_7 c_h)],$$

where A is the coefficient, which may be given an average value for each type of engine. The pressure left in the receiver is P_8, and may be found by putting A equal to unity. This is the pressure when the high-pressure exhausts into the receiver, so that

$$p_2 (\mathrm{H} + c_h) + \mathrm{P}_8 \mathrm{R} = p_3 (\mathrm{H} + c_h + \mathrm{R}).$$

Neglecting the obliquity of the connecting rod, supposing that admission to the low-pressure cylinder is instantaneous, the steam will be compressed to a pressure p_4 at half stroke of the high-pressure piston, and

$$p_4 (\tfrac{1}{2} \mathrm{H} + c_h + \mathrm{R}) = p_3 (\mathrm{H} + c_h + \mathrm{R}),$$

and

$$p_4 (\tfrac{1}{2} \mathrm{H} + c_h + \mathrm{R}) + p_{10} c_l = p_5 (\tfrac{1}{2} \mathrm{H} + c_h + \mathrm{R} + c_l),$$

so that the theoretical diagram would show a sudden drop in pressure. In the actual diagram this never appears, because admission to the low-pressure cylinder is gradual, and even if the port were opened suddenly the pressure in the two cylinders would not be suddenly equalised. A line from 3 to 5 agrees very closely with the actual diagram. In the actual diagram the point 5′ is below 5, because of the resistance of the passages between the cylinder; the difference is from 2 lb. to 5 lb. Compression still continues, because the high-pressure piston is moving rapidly and the low-pressure piston slowly. The curve 56 is best drawn by finding the volume V filled by the steam for any position of the high-pressure piston, and using the equation—

$$p . \mathrm{V} . = p_5 (\tfrac{1}{2} \mathrm{H} + c_h + \mathrm{R} + c\ ),$$

the corresponding positions of the two pistons, and therefore the volume filled can be found from fig. 195. If A B, C D be drawn at right angles, and A H E is the angle made by the high-pressure crank with the line of stroke, then C H E will be the angle made by the low-pressure crank. The portion of the high-pressure stroke to be completed is F B, and the low-pressure piston is G C from the commencement. If we draw from 7 a hyperbola whose asymptotes are *a b* and the line of no pressure, it will intersect this curve in the point 6, which gives the point of compression. If there were no fall of pressure between the two cylinders, the point 6′ would be on a level with 6 ; in practice it will be from 2 lb. to 5 lb. below this. After compression has commenced in the high-pressure cylinder, this generally taking place before cut-off in the low-pressure cylinder, the steam expands in the receiver and this cylinder, so that the curve 6′8 would be a hyperbola, with asymptotes *a b*, *b h*, if there were no wire-drawing. This hyperbola may be drawn through 6′ to a point close to 8′ when the pressure falls rapidly, as shown in fig. 194. The curve 89 is an hyperbola with *c d*, *d h* as asymptotes, the dotted lines at the right showing the exhaust.

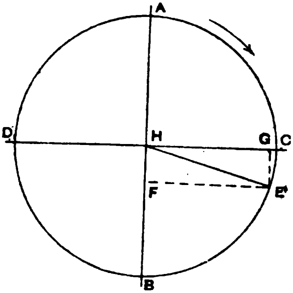


FIG. 195.

Next let us suppose cut-off after half stroke, fig. 196 ; then

$$p_2 (\mathrm{H} + c_h) = p_1 (k\,\mathrm{H} + c_h)$$

$$p'_8 (k_1\,\mathrm{L} + c_l) - p_{10}\, c_l = \mathrm{A}\,[p_1 (k\,\mathrm{H} + c_h) - p_6\, c_h]$$

The curve 8′9 is a hyperbola with *c d*, *d h* as asymptotes. At half-stroke the pressure suddenly increases in the low-pressure cylinder, because exhaust takes place in the high-pressure cylinder, and this for simplicity is supposed to be instantaneous, and at the end of the stroke. Point 8 is from 2 lb. to 5 lb. above 8′; while the high-pressure piston moves from 3 to 8 the low-pressure piston moves from 3′ to 8′—that is, from mid-stroke to cut-off. Let l be the fraction of the stroke of the high-pressure piston that remains when it has reached 8; then

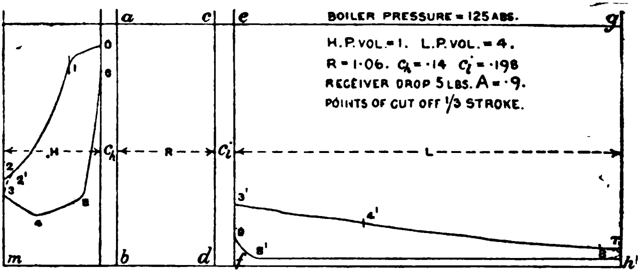

a
c
e
BOILER PRESSURE = 125 ABS.
g
H.P. VOL. = 1. L.P. VOL. = 4.
R = 1·06. c_h = ·14 c_l = ·198
RECEIVER DROP 5 LBS. A = ·9.
POINTS OF CUT OFF 1/3 STROKE.
0
6
1
H
c_h
R
c_l
L
2
2′
3
8
4
3′
4′
9
8′
7
8
m
b
d
f
h′

FIG. 196.

$$p'_3\,(\mathrm{H} + c_h + \mathrm{R} + c_l + \tfrac{1}{2}\,\mathrm{L}) = p'_8\,(l\,\mathrm{H} + c_h + \mathrm{R} + c_l + k_1\,\mathrm{L}),$$

and point 3 may be drawn as much above 3′ as 8 is above 8′. Draw 84 a hyperbola, with $c\,d$, $d\,m$ asymptotes, so that

$$p_4\,(\tfrac{1}{2}\,\mathrm{H} + c_h + \mathrm{R}) = p_8\,(l\,\mathrm{H} + c_h + \mathrm{R}),$$

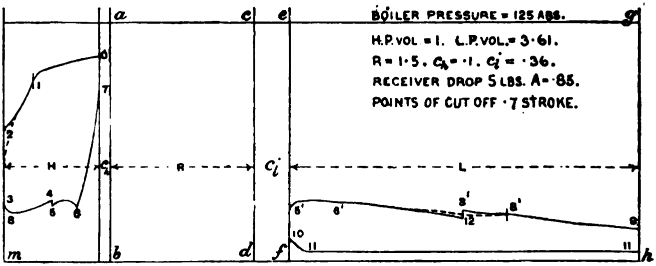

a
c
e
BOILER PRESSURE = 125 ABS.
g
H.P. VOL. = 1. L.P. VOL. = 3·61.
R = 1·5. c_h = ·1. c_l = ·36.
RECEIVER DROP 5 LBS. A = ·85.
POINTS OF CUT OFF ·7 STROKE.
0
1
7
2
H
c_h
R
c_l
L
3
4
5
6
8
5′
6′
3′
8′
12
9
10
11
11
m
b
d
f
h

FIG. 197.

and

$$p_5\,(\tfrac{1}{2}\,\mathrm{H} + c_h + \mathrm{R} + c_l) = p_4\,(\tfrac{1}{2}\,\mathrm{H} + c_h + \mathrm{R}) + p_{10}\,c_l.$$

A line from 8 to 5 will agree closely with practice. We must next take 5′ and 6′ from 2 lb. to 5 lb. below 5 and 6, and we may find p_6 thus: Draw a hyperbola 76, whose asymptotes are $a\,b$, $b\,m$, until it intersects the curve 56, drawn as explained in the last case; lastly, 12.6′ is a hyperbola having $a\,b$, $b\,h$ for its asymptotes.

A third case is shown in fig. 197, when the arrangement of cylinders is what is called "tandem," or, when the two pistons are connected to two cranks at 180 deg., the diagrams take this form,

$$p_2\,(\mathrm{H} + c_h) = p_1\,(k\,\mathrm{H} + c_h)$$

$$p'_4\,(k_1\,\mathrm{L} + c_l) - p_9\,c_l = \mathrm{A}\,[p_1\,(k\,\mathrm{H} + c_h) - p_6\,c_h]$$

where A is a coefficient generally less than unity; p_4 is from 2 lb. to 5 lb. more than p'_4, and may be calculated by the equation

$$p_4\,(k_1\,\mathrm{L} + c_l) - p_9\,c_l = p_1\,(k\,\mathrm{H} + c_h) - p_6\,c_h$$

$$p_3\,(\mathrm{H} + c_h + \mathrm{R} + c_l = p_4\,[(1 - k_1)\,\mathrm{H} + c_h + \mathrm{R} + c_l + k_1\,\mathrm{L}].$$

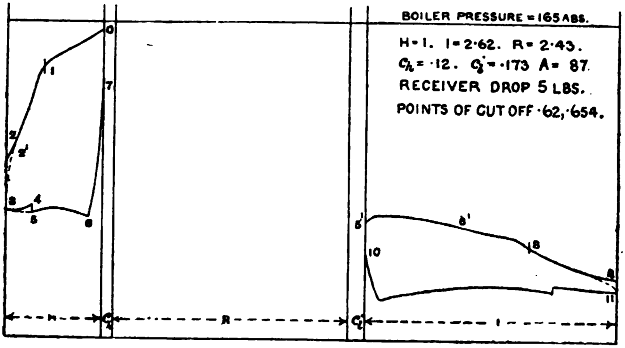

BOILER PRESSURE = 165 ABS.
H = 1. l = 2·62. R = 2·43.
c_h = ·12. c_l = ·173 A = 87.
RECEIVER DROP 5 LBS.
POINTS OF CUT OFF ·62, ·654.
0
1
7
2
2′
3
4
5
6
5′
6′
10
8
9
11
H
c_h
R
c_l
l

FIG. 198.

Point 5 may be found by drawing two intersecting hyperbolæ, 65 and 45, with asymptotes $a\,b$, $b\,m$ and $c\,d$, $d\,m$ respectively, because the former is the curve of compression of steam in the high-pressure cylinder, and the latter of steam in high-pressure cylinder and receiver after cut-off in the low-pressure cylinder. We must take 3′ from 2 lb. to 5 lb. below 3.

In triple-expansion engines with three cranks at 120 deg. the sequence of cranks may be low, intermediate, high, or the reverse. These two arrangements are called low and high pressure cranks leading. The former arrangement gives less range of temperature and initial stress in each cylinder, but in spite of these advantages there are many engineers who prefer the latter arrangement. Fig. 198 shows the high and intermediate pressure diagrams with low-pressure crank leading. In fig. 199 A B and C D are two diameters at 120 deg. By drawing perpendiculars E F, E G from any point E on the circle, corresponding points of the strokes of high-pressure and intermediate pistons are found, the obliquity of the connecting rod being neglected. When E is at B the exhaust from the high-pressure cylinder is taking place, and admission to the intermediate does not commence until E has reached C and the high-pressure piston has made one-quarter of its return stroke. Compression takes place during this period in the high-pressure cylinder and receiver, and this is shown by the line 34. There is a slight drop when admission to the intermediate-pressure cylinder commences, and the pressure then rises in both cylinders, because the high-pressure piston is moving with considerable velocity and the intermediate-pressure piston very slowly at first. If there were no resistance to the flow of steam through the passages, the pressure would be greatest when the areas of the two pistons, multiplied by their respective velocities, become equal to one another. When compression commences in the high-pressure cylinder expansion continues in the receiver and intermediate-pressure cylinder until the point of cut-off, which we shall assume to be not later than three-quarter stroke, so that the exhaust from the high-pressure cylinder, supposed to take place instantaneously and at the end of the stroke,

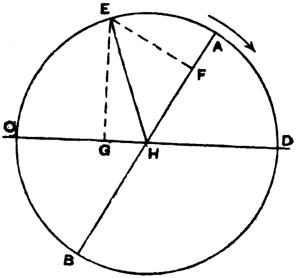


FIG. 199.

occurs after cut-off in the intermediate-pressure cylinder. The equations are—

$$p_1 (k\mathrm{H} + c_h) = p_2 (\mathrm{H} + c_h)$$

$$p_8 (k_1 \mathrm{I} + c_i) - p_{10}\, c_i = \mathrm{A}[p_1 (k\mathrm{H} + c_h) - p_7\, c_h]$$

where I and c_i are the volumes of the intermediate cylinder and its clearance, and k_1 is the fraction of the stroke at which cut-off takes place. If there were no wire-drawing, the pressure in the receiver at the moment of exhaust from the high-pressure cylinder would be p_8, but it is larger than this, and may be calculated by the equation—

$$\mathrm{P}_8 (k_1 \mathrm{I} + c_i) - p_{10}\, c_i = p_1 (k\mathrm{H} + c_h) - p_7\, c_h$$

and

$$p_2 (\mathrm{H} + c_h) + \mathrm{P}_8 \mathrm{R} = p_3 (\mathrm{H} + c_h + \mathrm{R})$$

$$= p_4 (\tfrac{3}{4}\mathrm{H} + c_h + \mathrm{R})$$

$$\mathrm{P}_4 (\tfrac{3}{4}\mathrm{H} + c_h + \mathrm{R}) + p_{10}\, c_i = p_5 (\tfrac{3}{4}\mathrm{H} + c_h + \mathrm{R} + c_i).$$

The curves 5 6, 5′ 6′ are found by construction, as in fig 194, corresponding points in the latter being taken from 2 lb. to 5 lb. below those in the former. The curve 6′ 8 would be part of a hyperbola, because expansion is taking place in the receiver and intermediate-pressure cylinder, but wire-drawing makes it drop rapidly just before cut-off. The pressures during the intermediate-pressure exhaust stroke and low-pressure admission may be found in a manner similar to the above; a coefficient B must be used, in the same manner as A above, in the equation containing the pressure at cut-off in the low-pressure cylinder.

When the high-pressure crank leads, the relative positions of the high-pressure and intermediate-pressure pistons is shown by fig. 199, the direction of rotation being opposite to that shown by the arrow, so that when the high-pressure exhaust stroke commences the intermediate-pressure piston is at quarter-stroke, and the pressure rises from 9 to 3′, fig. 200. The steam then expands in both cylinders and the receiver until steam is cut off in the intermediate-pressure cylinder, when compression takes place in the high-pressure cylinder and receiver until three-quarter stroke; when steam is admitted to the other side of the intermediate-pressure piston, there is a sudden fall of pressure from 5 to 6, and steam is first compressed and afterwards expanded in both cylinders and the receiver until compression commences in the high-pressure cylinder, when the steam in the receiver and intermediate-pressure cylinder expands until quarter-stroke. In the actual diagrams the sudden changes of pressure 56 and 93′ do not appear, and a line drawn from 4 to 6 will agree closely with the actual diagram, and the actual curve of admission may be drawn about midway between 9 and 3′, allowance being made for drop of pressure between the two cylinders.

The equations are as follow :—

$$p'_4\,(k_1\,\mathrm{I} + c_i) - p_{12}\,c_i = \mathrm{A}\,[p_1\,(k\,\mathrm{H} + c_h) - p_8\,c_h].$$

p_4 may be found from the above by putting A = to unity. We can find from fig. 199 the volume V between the high-pressure and intermediate-pressure pistons, and since from

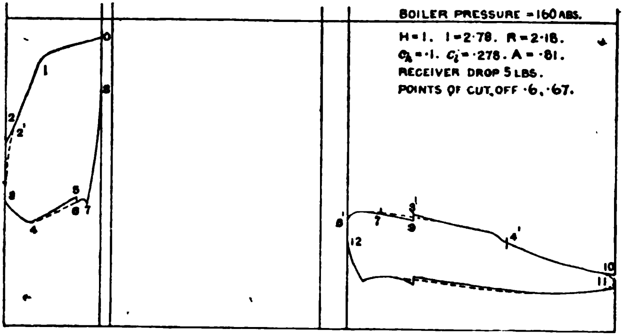

BOILER PRESSURE = 160 ABS.
H = 1. I = 2·78. R = 2·18.
c_h = ·1. c_i = ·278. A = ·81.
RECEIVER DROP 5 LBS.
POINTS OF CUT.OFF ·6, ·67.
0
1
8
2
2′
3
5
6
7
4
8′
7
3′
9
4′
12
10
11

FIG. 200.

3 to 4 the steam is expanding in the two cylinders and the receiver—

$$p_3\,(\mathrm{H} + c_h + \mathrm{R} + c_i + \tfrac{1}{4}\,\mathrm{I}) = p_4\,\mathrm{V},$$

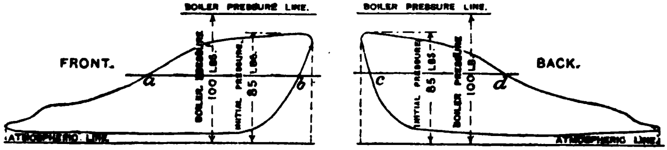
and 3′ may be taken from 2 lb. to 5 lb. below 3. From 4 to 5 compression takes place in the high-pressure cylinder and receiver, so that, if V_1 is the volume filled by the steam at 4,

$$p_4\,\mathrm{V}_1 = p_5\,(\tfrac{1}{4}\,\mathrm{H} + c_h + \mathrm{R}),$$

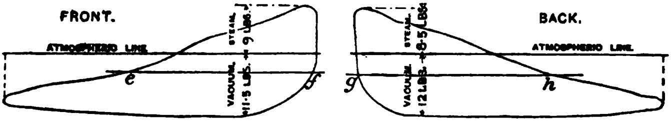
and $p_6\,(\tfrac{1}{4}\,\mathrm{H} + c_h + \mathrm{R} + c_i) = p_5\,(\tfrac{1}{4}\,\mathrm{H} + c_h + \mathrm{R}) + p_{12}\,c_i.$

The curve 67 may be found in the same way as 56, fig. 194, and 7′ 9 is part of a hyperbola, expansion taking place in the receiver and intermediate-pressure cylinder until quarter-stroke. The low-pressure diagram and the exhaust line of the intermediate-pressure diagram may be found in a similar manner; a coefficient B must be used in the same manner as A above in the equation containing the pressure at cut-off in the low-pressure cylinder.

The following is the method of finding from actual diagrams the value of A and B. The diagrams, fig. 201, are taken from *The Practical Engineer* of July 28th, 1893. They are those of a horizontal tandem mill engine, cylinders 21 in. and 42 in. diameter, and 5 ft. stroke, so that the high-pressure volume is one-quarter that of the low-pressure cylinder. Lines *a b*, *c d*, *e f*, *g h* are drawn parallel to the atmospheric line, so that they cut the expansion and compression curves. The length of the front diagram is



HIGH-PRESSURE CYLINDER.



LOW-PRESSURE CYLINDER.

FIG. 201.

3·17 in., and *a b* is 1·59 in., and the pressure at *a* or *b* is 66·75 absolute. Here, then, the volume represented by *a b* is

$$V = \frac{a\,b\,.\,H}{M} = \frac{1{\cdot}59}{3{\cdot}17}\,H,$$

where M is the length of diagram and H the volume of the high-pressure cylinder. If, then, we use the figures in fig. 143, and if p is the pressure at *a* or *b*, then

$$p\,.\,V = p_1\,(k\,H + c_h) - p_6\,c_h)$$
$$= 66{\cdot}75 \times \frac{1{\cdot}59}{3{\cdot}17}\,H = 33{\cdot}5\,H.$$

Treating the back diagram in the same way, we find

$$p\,V = 30{\cdot}4\ H.$$

The mean value is

$$31{\cdot}95\ H.$$

In the low-pressure front diagram, if P is the pressure at *ef*, and v the volume represented by that line,

$$\mathrm{P}v = p^1{}_4 (k_1 \mathrm{L} + cl) - p_9\, cl.$$

The length of this diagram is 3·22 in., *e f* is 1·89, and

$$\mathrm{P} = 11{\cdot}17$$

absolute, so that

$$\mathrm{P}v = \frac{11{\cdot}17 \times 1{\cdot}89}{3{\cdot}22} \times \mathrm{L} = 26{\cdot}3\,\mathrm{H},$$

since

$$\mathrm{L} = 4\,\mathrm{H}.$$

The same quantity for the back diagram is 26·25 H. The mean value of P v is therefore 26·275 H ; but

$$\mathrm{A} = \frac{\mathrm{P}v}{p\mathrm{V}} = \frac{26{\cdot}275}{31{\cdot}95} = {\cdot}822.$$

If no line can be drawn cutting both compression and expansion curves, we cannot use the diagrams for this purpose unless the clearance volumes are known, which is not often the case ; knowing these, the values of p V Pv can be calculated by taking two points, one on the expansion curve and another on the compression curve, in each diagram ; the product of pressure and volume on the former is p_1 $(k\mathrm{H} + ch)$ on the high-pressure diagram, and on the latter is $p_6\, ch$, so that the difference of these two is p V. Again, in the low-pressure diagram we can obtain from the points on expansion and compression curves $p_8\, (k_1 \mathrm{L} + cl\,\rangle - p_{10}\, cl.$, fig. 194, which equals Pv. Thus in the mean diagrams of the "Colchester's" compound inverted screw engine with cranks at right angles, tested by Professor Kennedy, we have—

$$\mathrm{L} = 3{\cdot}61\,\mathrm{H}, \quad ch, = {\cdot}0939\,\mathrm{H}, \quad cl = {\cdot}0623\,\mathrm{L}.$$

The compression pressure in the high-pressure cylinder is 37·2 and in the low-pressure cylinder 10 absolute.

At 70 per cent of high-pressure stroke the pressure is 63·2, so that we obtain—

$$p_1\,(k\,\mathrm{H} + ch) = 63{\cdot}2 \times {\cdot}7939\,\mathrm{H} = 50{\cdot}15\,\mathrm{H}$$

$$p_7\, ch = 37{\cdot}2 \times {\cdot}0939\,\mathrm{H} = 3{\cdot}49\,\mathrm{H}\,;$$

$$\therefore p\,\mathrm{V} = 46{\cdot}66.$$

At 80 per cent of the low-pressure stroke the pressure is 13·7; whence—

$$p_8(k_1 \text{L} + cl) = 13{\cdot}7 \times {\cdot}8623\,\text{L} = 42{\cdot}65\,\text{H}$$

$$p_{10}\,cl = 10 \times {\cdot}0623\,\text{L} = 2{\cdot}25\,\text{H};$$

$$\therefore \text{P}\,v = 40{\cdot}4$$

$$\text{A} = \frac{40{\cdot}4}{46{\cdot}66} = {\cdot}869.$$

An average value of A for compound engines is ·85, and for triple-expansion engines A and B may be ·89 and ·77 respectively, these being the means of a large number of examples.

It is impossible to give any fixed rules for the drops from boiler to initial and cut-off pressures in the high-pressure cylinder, as these depend on piston speed, area of passages and point of cut-off, distance of boiler from engine, and bends in the steam pipes. In a large number of cases from triple-expansion engines it was between 4 lb. and 16 lb. from boiler to admission, and the mean value was a little over 10 lb.; the drop from boiler pressure to cut-off was from 23 lb. to 37 lb., and the mean value a little over 28 lb. In the diagrams of compound engines, with slide valves, the above mean values were 12 lb. and 23 lb., and varied between 5 lb. and 19 lb. for the first, and 19 lb. and 30 lb. for the second. The boiler pressure for the triples was between 150 lb. and 170 lb.; and for the compounds from 60 lb. to 110 lb.

Examples of diagrams of Corliss engines may be found in D. K. Clark's "Steam Engine"; and of compound locomotives in several volumes of the Proceedings of the Institution of Mechanical Engineers.

To find the size of low-pressure cylinder, we may proceed in the following manner:—

Suppose we require a triple-expansion marine engine to develop 850 horse power, with a piston speed of 450 ft. per minute, and 75 revolutions, the boiler pressure being 150 lb. This will give us a stroke of 3 ft., and the terminal pressure in the low-pressure cylinder should be about 10 lb.

But it is clear that if the terminal pressure is p, and the cut-off pressure in the high-pressure cylinder is p_1, that the number of expansions is

$$r = \frac{p_1}{p} \times \text{A}\,\text{B}$$

$$= \frac{p_1}{p} \times {\cdot}89 \times {\cdot}77 = \frac{p_1}{p} \times {\cdot}686.$$

Taking a mean value of drop—

$$p_1 = 165 - 28 = 137 \text{ lb.}$$

$$\therefore r = 13{\cdot}7 \times {\cdot}686 = 9{\cdot}3 \text{ lb.}$$

We must now assume an average diagram factor k. Then, if p_e be the mean effective pressure referred to the low-pressure piston—

$$\text{H.P.} = \frac{p_e\, l\, a\, n}{33000}$$

and $$p_e = k\left\{\text{P}\,\frac{1 + \log r}{r} - 3\right\}$$

where P is the absolute pressure in the boiler, and 3 is subtracted for back pressure. A mean value of η is ·6, so that we find—

$$p_e = {\cdot}6 \times \}\,165 \times {\cdot}348 - 3\,\{ = 32{\cdot}6.$$

$$\therefore a = \frac{850 \times 33000}{32{\cdot}6 \times 450} = 1910.$$

This corresponds to a diameter of 49⅜ in. We can, if we prefer it, find the volume swept out by the low-pressure piston per minute before fixing on the length of stroke and number of revolutions. The cut-off in the high-pressure cylinder lies between 60 and 75 per cent of the stroke. Suppose we assume a cut-off of 65 per cent, and a is the area of the high-pressure piston.

$${\cdot}65\, a_1 = \frac{a}{r} \quad \therefore a_1 = \frac{1910}{{\cdot}65 \times 9{\cdot}3} = 316 \text{ square inches,}$$

so that the diameter of piston will be 20 in. The diameters of the three pistons are in geometrical progression, and the same applies to the four piston diameters of a quadruple-expansion engine, so that if d_2 is the diameter of the intermediate piston,

$$d_2 = \sqrt{20 \times 49\tfrac{3}{8}} = 31{\cdot}4 \text{ in.}$$

The points of cut-off in the intermediate and low pressure cylinders may then be assumed at about 60 per cent of the stroke, and the probable diagrams can now be drawn. In triple-expansion marine engines with three cranks the first receiver is from two or three times the volume of the high-pressure cylinder, and the second is from ·9 to 1½ times the volume of the intermediate cylinder in the examples we have calculated. It is generally a very tedious process calculating these volumes, and it is rarely done, which is our only apology for not giving fuller information on this point.

In order to show the accuracy of this method of setting out diagrams, it would be necessary to give a very large number of examples of comparison of actual with theoretical diagrams. This would take up too much space; we have therefore selected the diagrams of a triple-expansion marine engine given by Mr. J. P. Hall in his paper, "Compound *versus* Triple-expansion Engines," read before the North-East Coast Institution of Engineers and Shipbuilders, in 1887.

The low-pressure crank leads, the cylinders are 21 in., 34 in., 57 in. in diameter, with 39 in. stroke; the boiler pressure is 165 lb. absolute; the points of cut-off in the three cylinders are at ·62, ·654, and ·595 of the strokes.

$$\mathrm{I} = 2{\cdot}62\,\mathrm{H}\,; \qquad \mathrm{H} = {\cdot}382\,\mathrm{I}\,;$$

$$\mathrm{L} = 2{\cdot}81\,\mathrm{I}\,; \qquad \mathrm{I} = {\cdot}356\,\mathrm{L}\,;$$

$$c_h = {\cdot}12\,\mathrm{H}\,; \; c_i = {\cdot}066\,\mathrm{I} = {\cdot}173\,\mathrm{H}\,;$$

$$c_l = {\cdot}064\,\mathrm{L} = {\cdot}18\,\mathrm{I}.$$

The three last are taken from the combined diagram given by Mr. Hall, which we have not reproduced here, as there is no necessity to do so. The volumes of the two receivers R and R_1 are not given in the paper, but Mr. Hall gives them as—

$$\mathrm{R} = 35154 \text{ cubic inches} = 2\,605\,\mathrm{H} = {\cdot}994\,\mathrm{I}.$$

$$\mathrm{R}_1 = 39100 \text{ cubic inches} = 1{\cdot}105\,\mathrm{I} = {\cdot}394\,\mathrm{L}.$$

In fig. 5 the high-pressure and intermediate-pressure diagrams only are given. In the following calculations we shall refer to p_{13}, the pressure at one-quarter stroke during the exhaust from the intermediate-pressure cylinder just before admission takes place to the low-pressure cylinder; p_{14}, p'_{14}, the pressures just after the admission, the former at one-quarter stroke of the intermediate-pressure piston, the latter at the commencement of the low-pressure stroke; p_{16}, p^{1}_{16} are the pressures in intermediate-pressure and low-pressure cylinders when compression commences in the former; p_{17} is the pressure at cut-off in the low-pressure cylinder. The calculations are as follow:—

$$p_8\,({\cdot}654\,\mathrm{I} + {\cdot}066\,\mathrm{I}) - p_{10} \times {\cdot}066\,\mathrm{I}.$$

$$= p_1\,(62 + {\cdot}12)\,\mathrm{H} - p_7 \times {\cdot}12\,\mathrm{H}.$$

Assuming p_1, p_7, p_{10} as 142 lb., 103 lb., and 43 lb., their actual values, we obtain

$$p_8 = 53{\cdot}1.$$

Its actual value at cut-off in the intermediate-pressure cylinder is 46·6, which will be obtained if we put

$$A = {\cdot}866.$$

We assume that the pressure in the receiver is

$$P_8 = 53{\cdot}1 \text{ lb.},$$

whence

$$p_3 (H + c_h + R) = p_1 ({\cdot}62\, H + c_h) + P_8 \,.\, R.$$

$$p_3 = \frac{142 \times {\cdot}74 + 53{\cdot}1 \times 2{\cdot}605.}{3{\cdot}725.} = 65{\cdot}4.$$

Actual pressure = 63·2 ; difference = 2·2.

$$p_4 ({\cdot}75\, H + c_h + R) = p_3 (H + c_h + R)$$

$$p_4 = 70.$$

Actual pressure = 69 lb. ; difference = 1.

$$p_5 ({\cdot}75\, H + c_h + R + c_i) = p_4 ({\cdot}75\, H + c_h + R) + p_{10}\, c_i$$

$$p_5 = 68{\cdot}6.$$

Actual pressure = 69 ; difference = − ·4 lb.

Actual $p^1{}_5$ = 65 ; difference = 3·6 lb.

Compression commences at ·90 of the H.P. exhaust stroke when the I.P. piston has made ·441 of its stroke.

$$\therefore\ p_6 ({\cdot}1\, H + c_h + R + c_i + {\cdot}441\, I)$$

$$= p_5 ({\cdot}75\, H + c_h + R + c_i)$$

$$p_6 = 60{\cdot}25.$$

Actual value 64·5 ; difference = − 4·25.

In the I.P. cylinder

$p^1{}_6$ = 54·5 ; difference 6·25 lb.

$$p_7\, c_h = p_6 ({\cdot}1\, H + c_h)$$

$$p_7 = 110$$

Actual pressure = 103 ; difference 7 lb.

This shows very close agreement between theory and practice, if we allow a drop of 5 lb. between the two cylinders.

At cut-off in the low-pressure cylinder

$$p_{17}(\cdot 595\,\mathrm{L} + c_l) - p_{20}\,c_l$$
$$= p_8 \times \cdot 72\,\mathrm{I} - p_{10}\,c_i.$$

p_{20} is 7·8 lb., whence

$$p_{17} = 17\cdot 5.$$

Actual value 13·1 ; difference 4·4.

We assume, then, that 17·5 is the pressure in the second receiver, and is represented by P_{17}.

We can obtain the actual value of p_{17} by making B equal to ·75, very nearly.

$$p_8(\cdot 654\,\mathrm{I} + c_i) + \mathrm{R}_1\,\mathrm{P}_{17} = p_{11}(\mathrm{I} + c_i + \mathrm{R}_1)$$

$$46\cdot 6 \times \cdot 72 + 1\cdot 105 \times 17\cdot 5 = p_{11} \times 2\cdot 171$$

$$p_{11} = 24\cdot 3.$$

Actual pressure = 23·75 ; difference = ·55.

$$p_{13} = \frac{p_{12}(\mathrm{I} + c_i + \mathrm{R}_1)}{\frac{3}{4}\,\mathrm{I} + c_i + \mathrm{R}_1} = \frac{52\cdot 85}{1\cdot 921.}$$
$$= 27\cdot 5.$$

Actual pressure = 25·7 ; difference = 1·8.

$$p_{14}(c_l + \cdot 75\,\mathrm{I} + c_i + \mathrm{R}_1) = p_{13}(\cdot 75\,\mathrm{I} + c_i + \mathrm{R}_1) + p_{20}\,c_l$$

$$p_{14} = 25\cdot 85.$$

Actual pressure = 25·7 ; difference ·15.

Actual $p^1{}_{14}$ = 20·7 ; difference = 5·15.

Compression in the intermediate-pressure cylinder takes place at nine-tenths of the exhaust stroke when the low-pressure piston has made ·44 of its forward stroke.

$$p_{16}(\cdot 1\,\mathrm{I} + c_i + \mathrm{R}_1 + c_l + \cdot 44\,\mathrm{L})$$
$$= p_{14}(c_i + \cdot 75\,\mathrm{I} + c_l + \mathrm{R}_1)$$
$$p_{16} = 20\,2.$$

Actual pressure = 20·5 ; difference = − ·3.

Actual $p^{1}{}_{16}$ = 14·2 ; difference = 6.

$$p_{10}\,c_i = p_{16}\,(1\,\mathrm{I} + c_i).$$

$$p_{10} = \frac{20{\cdot}2 \times {\cdot}166}{{\cdot}066}$$

$$= 50{\cdot}8 \text{ lb.}$$

p_{10} is assumed as 43 lb. ; difference = 7·8.

The agreement is, therefore, very close on the whole, except for p_7 and p_{10}, and in drawing the diagrams a mean can here be taken between the assumed and calculated values—that is, if we assume these values first, and also assume the points of compression. We have already shown that the point of compression may be found by construction; but this is more tedious than the above method of calculation.

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